

E. Long Answer

★ 5 MARKS

Q1 CBSE 2026

Let p be a prime number. Show that the relation $R = \{(x, y) : |x - y| \text{ is divisible by } p\}$ on $\mathbb{Z}$ is an equivalence relation.

Solution:

Reflexive: $|x - x| = 0 = p \cdot 0$ is divisible by p , so $(x, x) \in R$ for every $x \in \mathbb{Z}$.

Symmetric: $|x - y| = |y - x|$, so if $|x - y|$ is divisible by p so is $|y - x|$. Hence $(x, y) \in R \Rightarrow (y, x) \in R$.

Transitive: Let $|x - y| = pk$ and $|y - z| = pm$ (with $k, m \geq 0$ integers). Then $x - y = \pm pk$ and $y - z = \pm pm$, so $x - z = (x - y) + (y - z) = p(\pm k \pm m)$, a multiple of p . Hence $|x - z|$ is divisible by p and $(x, z) \in R$.

Since R is reflexive, symmetric and transitive, it is an **equivalence relation**.

✓ **Answer:** Equivalence relation.

Q2 CBSE 2026

Show that $f: R - \left\{\frac{3}{5}\right\} \rightarrow R - \left\{\frac{3}{5}\right\}$, $f(x) = \frac{3x+2}{5x-3}$ is bijective. Also find f^{-1} .

Solution:

One-one: let $f(x_1) = f(x_2)$, i.e. $\frac{3x_1+2}{5x_1-3} = \frac{3x_2+2}{5x_2-3}$.

Cross-multiplying, $(3x_1 + 2)(5x_2 - 3) = (3x_2 + 2)(5x_1 - 3)$. The x_1x_2 and constant terms cancel, leaving $(-19)(x_1 - x_2) = 0$. Since $-19 \neq 0$, $x_1 = x_2$. So f is **one-one**.

Onto: let $y = 3x + 25x - 3$ be any value with $y \neq \frac{3}{5}$. Then $y(5x - 3) = 3x + 2$, so $x(5y - 3) = 3y + 2$, giving $x = \frac{3y+2}{5y-3}$, which exists because $5y - 3 \neq 0$.

This x is never equal to $\frac{3}{5}$ (that would force $bc = ad$, i.e. $10 = -9$, which is false), so x lies in the domain and $f(x) = y$. Hence f is **onto**.

From the onto step, $x = \frac{3y+2}{5y-3}$, so $f^{-1}(y) = \frac{3y+2}{5y-3}$, i.e. $f^{-1} = f$.

✓ **Answer:** f is bijective and

$$f^{-1}(x) = \frac{3x+2}{5x-3}.$$

Q3 CBSE 2026

Let R^+ be the set of non-negative real numbers. Show that $f: R^+ \rightarrow [-5, \infty)$, $f(x) = 4x^2 + 4x - 5$ is invertible. Find f^{-1} , and hence find the value of p for which $f(p) = 3$.

Solution:

Write $f(x) = (2x + 1)^2 - 6$.

One-one: for $x_1, x_2 \geq 0$, $f(x_1) = f(x_2) \Rightarrow (2x_1 + 1)^2 = (2x_2 + 1)^2$. Both $2x_i + 1$ are positive, so $2x_1 + 1 = 2x_2 + 1$ and $x_1 = x_2$.

Onto: for $y \geq -5$ let $x = \sqrt{y + 6} - 12$.

As $y + 6 \geq 1$, $\sqrt{y + 6} \geq 1$ so $x \geq 0$, and

$$f(x) = (\sqrt{y + 6})^2 - 6 = y.$$

So f is bijective, hence invertible, with

$$f^{-1}(y) = \sqrt{y + 6} - 12.$$

If $f(p) = 3$ then $p = f^{-1}(3) = \sqrt{9} - 12 = 1$.

Check: $f(1) = 4 + 4 - 5 = 3$.

✓ **Answer:** $f^{-1}(y) = \sqrt{y + 6} - 12$ and $p = 1$.

Q4 CBSE 2026

Let R be the relation on $N \times N$ defined by $(a, b) R (c, d)$ if and only if $ad = bc$. Show that R is an equivalence relation.

Solution:

Reflexive: $(a, b) R (a, b)$ needs $ab = ba$, which is true.

Symmetric: if $(a, b) R (c, d)$ then $ad = bc$, i.e. $cb = da$, which says $(c, d) R (a, b)$.

Transitive: let $(a, b) R (c, d)$ and $(c, d) R (e, f)$, so $ad = bc$ and $cf = de$.

Multiplying, $ad \cdot cf = bc \cdot de$.

Cancel cd (non-zero): $af = be$, so $(a, b) R (e, f)$.

Hence R is reflexive, symmetric and transitive: an **equivalence relation**.

✓ **Answer:** Equivalence relation.

Folding and composites (M3)

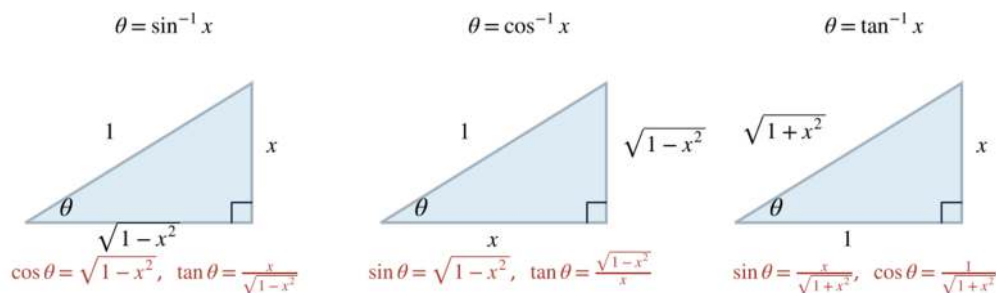
From the definition of an inverse: $\sin(\sin^{-1} x) = x$ for $x \in [-1, 1]$, and $\sin^{-1}(\sin \theta) = \theta$ only for $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. The first is always safe when x is in the domain; the second needs a check on θ . When θ is outside the branch, **fold** it back using 2π -periodicity and an allied angle:

Expression	Value of θ in ...	Result
$\sin^{-1}(\sin \theta)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	θ
	$\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$	$\pi - \theta$
	$\left[\frac{3\pi}{2}, \frac{5\pi}{2}\right]$	$\theta - 2\pi$
$\cos^{-1}(\cos \theta)$	$[0, \pi]$	θ
	$[\pi, 2\pi]$	$2\pi - \theta$
	$[2\pi, 3\pi]$	$\theta - 2\pi$
$\tan^{-1}(\tan \theta)$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	θ
	$\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$	$\theta - \pi$
	$\left(\frac{3\pi}{2}, \frac{5\pi}{2}\right)$	$\theta - 2\pi$
$\cot^{-1}(\cot \theta)$	$(0, \pi)$	θ
	$(\pi, 2\pi)$	$\theta - \pi$
	$(-\pi, 0)$	$\theta + \pi$

The result always lies in the branch. A quick check of any folding answer: is it inside $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (for $\sin^{-1}$) or $[0, \pi]$ (for $\cos^{-1}$)? If not, it is wrong.

Composites with a trigonometric function outside. To find $\sin\left(\cos^{-1}\frac{3}{5}\right)$, put $\theta = \cos^{-1}\frac{3}{5}$ so $\cos \theta = \frac{3}{5}$ with $\theta \in [0, \pi]$, then find $\sin \theta$ from a right triangle or $\sin \theta = \sqrt{1 - \cos^2 \theta}$ (positive, because $\theta \in [0, \pi]$).

The same triangles give the **conversions** below, which are used in nearly every simplification.



QUESTION BANK : SOLVED EXAMPLES (MCQS, ASSERTION-REASON, VSA, SA)

2. SOLVED EXAMPLES

A. MCQs

★ 1 MARK

Previous Year Questions

Q1 CBSE 2026

The number of all possible matrices of order 2×3 with each entry 1 or 2 is:

- (A) 16 (B) 81 (C) 64 (D) 32

✓ **Correct Answer:** (C) 64

Detailed Explanation: A 2×3 matrix has 6 entries; each entry can independently be 1 or 2 (2 choices), so the count is $2^6 = 64$.

Q2 CBSE 2026

Which of the following cannot be the order of a row matrix?

- (A) 2×1 (B) 1×2 (C) 1×1 (D) $1 \times n$

✓ **Correct Answer:** (A) 2×1

Detailed Explanation: A row matrix has exactly one row, so its order must be of the form $1 \times n$. The order 2×1 has two rows and one column — that describes a column matrix, not a row matrix.

Q3 CBSE 2026

Which of the following properties is/are true for two matrices A, B of suitable orders?

- (i) $(A + B)^T = A^T + B^T$
 (ii) $(A - B)^T = B^T - A^T$
 (iii) $(AB)^T = A^T B^T$
 (iv) $(kAB)^T = k B^T A^T$ (k scalar)

- (A) (i) only (B) (i), (ii) and (iii)
 (C) (i) and (ii) (D) (i) and (iv)

✓ **Correct Answer:** (D) (i) and (iv)

Detailed Explanation:

- (i) is the standard sum rule — true.
 (ii) is false; the correct identity is $(A - B)^T = A^T - B^T$. (iii) is false; the correct identity reverses the order: $(AB)^T = B^T A^T$.
 (iv) is true, since $(kAB)^T = k(AB)^T = k B^T A^T$.

Q4 CBSE 2026

If A and B are symmetric matrices of the same order, then which of the following is a skew-symmetric matrix?

- (A) $A^2 + B^2$ (B) $A^2 - B^2$
 (C) $AB + BA$ (D) $AB - BA$

✓ **Correct Answer:** (D) $AB - BA$

Detailed Explanation:

$(AB - BA)^T = B^T A^T - A^T B^T = BA - AB = -(AB - BA)$,
 since A and B are symmetric ($A^T = A, B^T = B$).
 So $AB - BA$ equals the negative of its own transpose — it is skew-symmetric.

Q5 CBSE 2026

If $A = [a_{ij}]$ is a 3×3 scalar matrix, then which of the following must be true?

- (A) A must be a symmetric matrix
 (B) A must be a skew-symmetric matrix
 (C) A must be an identity matrix
 (D) A must be a null matrix

✓ **Correct Answer:** (A) A must be a symmetric matrix

Detailed Explanation: A scalar matrix has $a_{ij} = k$ when $i=j$ and 0 when $i \neq j$, for some constant k. Since this pattern is symmetric in i and j regardless of the value of k, $A^T = A$ always — A is symmetric for every k, not only $k=1$ (identity) or $k=0$ (null).

B. Assertion–Reason

SOLUTIONS

Q1 Board Based

Assertion (A): If A and B are symmetric matrices of the same order, then AB is always symmetric.

Reason (R): The transpose of a product reverses order: $(AB)^T = B^T A^T$.

✓ **Correct Answer:** (D) A is false but R is true.

Explanation: R is a true, general fact about transposes. But the Assertion is false: $(AB)^T = B^T A^T = BA$ (using $A^T = A$, $B^T = B$), which equals AB only when $AB = BA$ — not guaranteed just because A and B are symmetric.

Q2 Board Based

Assertion (A): Every identity matrix is a scalar matrix.

Reason (R): A scalar matrix is a diagonal matrix whose diagonal entries are all equal to some constant k.

✓ **Correct Answer:** (A) Both A and R are true and R is the correct explanation of A.

Explanation: R gives the correct definition. The identity matrix is the special case $k=1$ of this definition, so it is indeed always a scalar matrix — confirming A, with R as the direct reason.

Q3 Competency Based

Assertion (A): If A is a 2×3 matrix and B is a 3×2 matrix, then AB and BA have the same order.

Reason (R): Matrix multiplication XY is defined whenever the number of columns of X equals the number of rows of Y.

✓ **Correct Answer:** (D) A is false but R is true.

Explanation: R correctly states the general rule for when a product is defined. But the Assertion is false: AB is 2×2 while BA is 3×3 — both products are

defined (as R explains) but they do not have the same order.

Q4 Board Based

Assertion (A): If A is a square matrix such that $A^2 = I$, then A is invertible.

Reason (R): A matrix A is invertible if some matrix B exists such that $AB = BA = I$.

✓ **Correct Answer:** (A) Both A and R are true and R is the correct explanation of A.

Explanation: R is the definition of invertibility. Since $A \cdot A = I$ directly, A itself plays the role of B in that definition (with $B = A$), so A is invertible — confirming A, with R as the exact reason.

Q5 HOTS · Think Beyond

Assertion (A): If A is a skew-symmetric matrix, then A^2 is symmetric.

Reason (R): $(A^2)^T = (A^T)^2 = (-A)^2 = A^2$

✓ **Correct Answer:** (A) Both A and R are true and R is the correct explanation of A.

Explanation: R computes $(A^2)^T$ correctly using $A^T = -A$, arriving back at A^2 . This shows A^2 equals its own transpose, i.e. symmetric — confirming A, with R as the proof.

Q6 Board Based

Assertion (A): The sum of a symmetric matrix and a skew-symmetric matrix of the same order is always symmetric.

Reason (R): Every square matrix can be written as the sum of a symmetric part and a skew-symmetric part.

✓ **Correct Answer:** (D) A is false but R is true.

Explanation: R states Theorem 2 correctly. But the Assertion is false: if P is symmetric and Q is skew-symmetric, $P+Q$ is just some general square matrix A (which is exactly what Theorem 2 decomposes) — it is symmetric only in the special case $Q=O$.

QUESTION BANK — SOLVED EXAMPLES (SECTIONS A–G: FULL CHAPTER)

2. SOLVED EXAMPLES

A. MCQs

★ 1 MARK

Previous Year Questions

Q1 CBSE 2026

If $A(\text{adj } A) = \begin{bmatrix} 2026 & 0 & 0 \\ 0 & 2026 & 0 \\ 0 & 0 & 2026 \end{bmatrix}$, then the value of $|\text{adj } A|$ is equal to:

- (A) 2026 (B) $(2026)^{-1}$
(C) $(2026)^{-2}$ (D) $(2026)^2$

✓ Correct Answer: (D) $(2026)^2$

Detailed Explanation: Since $A(\text{adj } A) = |A|I$, comparing with the given matrix gives $|A| = 2026$. For a 3×3 matrix, $|\text{adj } A| = |A|^{n-1} = |A|^2 = (2026)^2$.

Q2 CBSE 2026

If $A(\text{adj } A) = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then the value of $|2A|$ is:

- (A) 6 (B) 54 (C) 12 (D) 24

✓ Correct Answer: (D) 24

Detailed Explanation: $A(\text{adj } A) = |A|I$ gives $|A| = 3$ (order 3). For an $n \times n$ matrix, $|kA| = k^n |A|$, so $|2A| = 2^3 \times 3 = 8 \times 3 = 24$.

Q3 CBSE 2026

If A and B are square matrices of order 3 such that $|A| = 4$ and $|B| = 7$, then $|2AB|$ is:

- (A) 112 (B) 56 (C) 28 (D) 224

✓ Correct Answer: (D) 224

Detailed Explanation: $|2AB| = 2^3 |A| |B| = 8 \times 4 \times 7 = 224$.

Q4 CBSE 2026

If A and B are invertible matrices of the same order, then which of the following is NOT correct?

- (A) $(A + B)^{-1} = B^{-1} + A^{-1}$ (B) $(AB)^{-1} = B^{-1}A^{-1}$
(C) $\text{adj } A = |A|A^{-1}$ (D) $A^{-1-1} = A$

✓ Correct Answer: (A) $(A + B)^{-1} = B^{-1} + A^{-1}$

Detailed Explanation: (B), (C) and (D) are genuine identities: Theorem 4 gives (B); rearranging $A^{-1} = (1/|A|)\text{adj } A$ gives (C); and the inverse of an inverse returns the original matrix, giving (D). There is no such identity for the inverse of a sum — (A) is not correct in general.

Q5 CBSE 2026

A system of equations $AX = B$, where A is the coefficient matrix, is:

- (A) consistent if $|A| \neq 0$, with solution $X = BA^{-1}$
(B) inconsistent if $|A| = 0$ and $(\text{adj } A)B = O$
(C) inconsistent if $|A| \neq 0$
(D) possibly consistent or inconsistent if $|A| = 0$ and $(\text{adj } A)B = O$

✓ Correct Answer: (D) possibly consistent or inconsistent if $|A| = 0$ and $(\text{adj } A)B = O$

Detailed Explanation: $|A| \neq 0$ always gives a unique (hence consistent) solution $X = A^{-1}B$ — note the

⚠ Common Mistakes



MISTAKE

Checking only one of the two one-sided limits and assuming the other matches.



MISTAKE

Concluding "continuous, so differentiable" without a separate LHD=RHD check.



MISTAKE

Forgetting to multiply by $\frac{dy}{dx}$ when differentiating a term containing y in implicit differentiation.



MISTAKE

Using the power rule on x^x -type expressions instead of switching to logarithmic differentiation.



MISTAKE

Sign errors in the derivatives of $\cos^{-1} x$ and $\cot^{-1} x$ (forgetting the negative sign).



MISTAKE

In parametric differentiation, computing $\frac{dy}{dx}$ as $\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ — the ratio flipped.



MISTAKE

Forgetting the chain rule's "inner derivative" factor when differentiating a composite function.

⚠ Common Misconceptions

⚠ "If a function is continuous, it must also be differentiable."

What students commonly think: that "no breaks" (continuity) automatically means "smooth" (differentiable) — treating the two ideas as interchangeable.

Why that thinking is incorrect: differentiability is a STRICTER condition than continuity; a graph can have no break yet still have a sharp corner, where the slope approaching from the left differs from the slope approaching from the right.

Correct concept: differentiability $\Rightarrow$ continuity is a one-way implication. $f(x)=|x|$ is continuous everywhere but not differentiable at $x=0$, because LHD=-1 and RHD=+1 at that point — they disagree even though the function itself has no break.

How to avoid it in the exam: always run continuity and differentiability as two SEPARATE checks at a suspicious point (a junction of a piecewise function, an absolute-value expression), never assume one from the other.

⚠ "The derivative of a variable-exponent expression uses the ordinary power rule."

What students commonly think: that x^x or $(\sin x)^x$ can be differentiated the same way as x^n , by just bringing the exponent down.

Why that thinking is incorrect: the power rule nx^{n-1} only works when the exponent n is a CONSTANT; when the exponent itself is a function of x , neither the power rule nor the exponential rule ($a^x \ln a$) applies directly.

Correct concept: when both the base and the exponent are functions of x , take the logarithm of both sides first (logarithmic differentiation) to turn the expression into a sum/product that ordinary differentiation rules can handle.

Q6 CBSE 2026

The greatest integer function $f(x)=[x]$, $0 < x < 3$, is not differentiable at how many points?

- (A) At only one point
(B) At only two points
(C) At no point
(D) At three points

✓ **Correct Answer:** (B) At only two points

Detailed Explanation: $[x]$ fails to be differentiable at every integer (it isn't even continuous there). In the open interval $(0, 3)$, the integers are $x=1$ and $x=2$ — exactly 2 points.

Q7 CBSE 2026

If $f(x) = \frac{x^2 - 4x - 5}{x + 1}$ ($x \neq -1$), k ($x = -1$) is continuous at $x = -1$, then the value of k is:

- (A) any real value
(B) 6
(C) -1
(D) -6

✓ **Correct Answer:** (D) -6

Detailed Explanation: Factoring:

$x^2 - 4x - 5 = (x - 5)(x + 1)$, so $f(x) = x - 5$ for $x \neq -1$. $k = \lim_{x \rightarrow -1} (x - 5) = -6$.

Q8 CBSE 2026

The function $f(x) = \frac{3-x}{3x^2-x^3}$ is:

- (A) discontinuous at only one point
(B) discontinuous at exactly two points
(C) discontinuous at exactly three points
(D) continuous at every point

✓ **Correct Answer:** (B) discontinuous at exactly two points

Detailed Explanation: The denominator $3x^2 - x^3 = x^2(3 - x)$ is zero at $x=0$ and $x=3$, so f is undefined (hence discontinuous, since it fails to even be defined) at exactly these 2 points.

Q9 CBSE 2026

If $y = \log(\tan x)$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$ is:

- (A) $\frac{1}{\sqrt{2}}$ (B) 0 (C) 1 (D) 2

✓ **Correct Answer:** (D) 2

Detailed Explanation: $\frac{dy}{dx} = \frac{\sec^2 x}{\tan x} = \frac{2}{\sin 2x}$. At $x = \pi/4$: $\sin(\frac{\pi}{2}) = 1$, so $\frac{dy}{dx} = 2$.

Q10 CBSE 2025

The function f defined by $f(x) = x$ ($x \leq 1$), 5 ($x > 1$) is not continuous at:

- (A) $x=0$ (B) $x=1$ (C) $x=2$ (D) $x=5$

✓ **Correct Answer:** (B) $x=1$

Detailed Explanation: At $x=1$: LHL = $\lim_{x \rightarrow 1^-} x = 1 = f(1)$, but RHL = $\lim_{x \rightarrow 1^+} 5 = 5 \neq 1$. The one-sided limits disagree, so f is discontinuous at $x=1$.

Q11 CBSE 2025

If $f(x) = \frac{\sin^2(ax)}{x^2}$ ($x \neq 0$), 1 ($x=0$) is continuous at $x=0$, then the value of a is:

- (A) 1 (B) -1 (C) ± 1 (D) 0

✓ **Correct Answer:** (C) ± 1

Detailed Explanation:

Writing $\frac{\sin^2(ax)}{x^2} = a^2 \times \left[\frac{\sin(ax)}{ax} \right]^2$,

the limit as $x \rightarrow 0$ is $a^2 \times 1^2 = a^2$.

Setting $a^2 = 1$ gives $a = \pm 1$.

Q12 CBSE 2025

If $f(x)=[x]$ is the greatest integer function, then the correct statement about f at $x=2$ is:

- (A) f is continuous but not differentiable at $x=2$
(B) f is neither continuous nor differentiable at $x=2$
(C) f is continuous as well as differentiable at $x=2$
(D) f is not continuous but differentiable at $x=2$

Fresh Questions

Q3 Board Based

Assertion (A): $f(x)=|x|$ is differentiable at $x=0$.

Reason (R): $f(x)=|x|$ is continuous at $x=0$.

✓ **Correct Answer:** (D) A is false but R is true.

Explanation: R is true ($|x|$ has no break at 0). But the Assertion is false: $LHD=-1$ and $RHD=+1$ at $x=0$, so they disagree — $|x|$ is NOT differentiable at 0, even though it is continuous there.

Q4 Board Based

Assertion (A): If f is differentiable at $x=c$, then f is continuous at $x=c$.

Reason (R): Every differentiable function is continuous, though the converse need not hold.

✓ **Correct Answer:** (A) Both A and R are true and R is the correct explanation of A.

Explanation: R states the exact theorem, which directly confirms the Assertion — answer A.

Q5 Competency Based

Assertion (A): To differentiate $y=x^{\sin x}$, logarithmic differentiation must be used.

Reason (R): The ordinary power rule nx^{n-1} only applies when the exponent n is a constant.

✓ **Correct Answer:** (A) Both A and R are true and R is the correct explanation of A.

Explanation: R correctly explains why: since the exponent $\sin x$ is a function of x (not constant), neither the power rule nor a^x -style exponential rule applies directly, making logarithmic differentiation necessary — confirming A.

Q6 Board Based

Assertion (A): If $x=f(t)$ and $y=g(t)$, then $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

Reason (R): This follows directly from the chain rule applied to parametric equations.

✓ **Correct Answer:** (A) Both A and R are true and R is the correct explanation of A.

Explanation: R correctly identifies the source of the formula: the chain rule $dy/dt = (dy/dx)(dx/dt)$, rearranged, gives exactly the stated parametric formula — confirming A.

Q7 HOTS · Think Beyond

Assertion (A): A function that is continuous everywhere on $\mathbb{R}$ must be differentiable everywhere on $\mathbb{R}$.

Reason (R): $f(x)=|x|$ is continuous everywhere but not differentiable at $x=0$.

✓ **Correct Answer:** (D) A is false but R is true.

Explanation: R gives a true, standard counterexample. This directly disproves the Assertion, confirming it is false while R is a true statement.

C. Very Short Answer

★ 2 MARKS

Previous Year Questions

Q1 CBSE 2026

If $x=t+\frac{1}{t}$ and $y=t-\frac{1}{t}$, find $\frac{dy}{dx}$ at $t=2$.

Solution: $\frac{dx}{dt} = 1 - \frac{1}{t^2}$, $\frac{dy}{dt} = 1 + \frac{1}{t^2}$.

At $t=2$: $\frac{dx}{dt} = 1 - \frac{1}{4} = \frac{3}{4}$, $\frac{dy}{dt} = 1 + \frac{1}{4} = \frac{5}{4}$. $\frac{dy}{dx} = \frac{5/4}{3/4} = \frac{5}{3}$.

Q14 CBSE 2023

If $f(x)=a(x-\cos x)$ is strictly decreasing on $\mathbb{R}$, then a belongs to:

- (A) $\{0\}$ (B) $(0, \infty)$ (C) $(-\infty, 0)$ (D) $(-\infty, \infty)$

✓ **Correct Answer:** (C) $(-\infty, 0)$

Detailed Explanation: $f'(x)=a(1+\sin x)$.

Since $(1+\sin x) \geq 0$ always (ranging from 0 to 2), for $f'(x) \leq 0$ throughout (strictly decreasing, excluding $a=0$ which would make f constant), we need $a < 0$, i.e. $a \in (-\infty, 0)$.

Q15 CBSE 2023

The rate of change of the volume of a spherical bubble with respect to its radius r , at $r=3$ cm, is:

- (A) $24\pi \text{ cm}^3/\text{cm}$ (B) $36\pi \text{ cm}^2/\text{cm}$
(C) $36\pi \text{ cm}^3/\text{cm}$ (D) $24\pi \text{ cm}^2/\text{cm}$

✓ **Correct Answer:** (C) $36\pi \text{ cm}^3/\text{cm}$

Detailed Explanation: $V = \frac{4}{3}\pi r^3$, $\frac{dV}{dr} = 4\pi r^2$. At $r=3$:
 $\frac{dV}{dr} = 36\pi \text{ cm}^3/\text{cm}$.

Fresh MCQs

Q16 Board Based

A balloon's volume is increasing at $12 \text{ cm}^3/\text{s}$. The rate of change of its radius when $r=2$ cm is:

- (A) $\frac{3}{\pi} \text{ cm/s}$ (B) $\frac{3}{4\pi} \text{ cm/s}$ (C) $3\pi \text{ cm/s}$ (D) $\frac{12}{\pi} \text{ cm/s}$

✓ **Correct Answer:** (B) $\frac{3}{4\pi} \text{ cm/s}$

Detailed Explanation: $V = \frac{4}{3}\pi r^3$, so $\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$.
At $r=2$: $12 = 4\pi(4) \cdot \frac{dr}{dt}$, so $\frac{dr}{dt} = \frac{12}{16\pi} = \frac{3}{4\pi} \text{ cm/s}$.

Q17 Board Based

The function $f(x)=2x^3-3x^2-12x+5$ is strictly decreasing on:

- (A) $(-1, 2)$ (B) $(-\infty, -1)$ (C) $(2, \infty)$ (D) $\mathbb{R}$

✓ **Correct Answer:** (A) $(-1, 2)$

Detailed Explanation:

$f'(x)=6x^2-6x-12=6(x^2-x-2)=6(x-2)(x+1)$. This is

negative between the roots $x=-1$ and $x=2$, so f is strictly decreasing on $(-1, 2)$.

Q18 Board Based

The critical points of $f(x)=x^3-3x^2$ are:

- (A) $x=0, 2$ (B) $x=0, 3$ (C) $x=1, 2$ (D) $x=-1, 2$

✓ **Correct Answer:** (A) $x=0, 2$

Detailed Explanation: $f'(x)=3x^2-6x=3x(x-2)$.
Setting $f'(x)=0$: $x=0$ or $x=2$.

Q19 Competency Based

For $f(x)=x^3-3x^2$, the point $x=0$ is:

- (A) a point of local maximum
(B) a point of local minimum
(C) a point of inflexion
(D) not a critical point

✓ **Correct Answer:** (A) a point of local maximum

Detailed Explanation: $f'(x)=3x(x-2)$, which is positive for $x < 0$, negative for $0 < x < 2$. Since f' changes from $+$ to $-$ at $x=0$, this is a local maximum (First Derivative Test).

Q20 Board Based

If $f''(c)=0$ at a critical point c , the Second Derivative Test:

- (A) confirms c is a local maximum
(B) confirms c is a local minimum
(C) is inconclusive
(D) confirms c is neither max nor min

✓ **Correct Answer:** (C) is inconclusive

Detailed Explanation: When $f''(c)=0$, the Second Derivative Test gives no information either way — the First Derivative Test must be used instead to classify c .

Q21 Board Based

A ladder 5 m long is sliding down a wall. If the foot of the ladder is x m from the wall, and x is increasing, the rate of change of the height of the top of the ladder (given by y where $x^2+y^2=25$) with respect to

D. Short Answer

★ 3 MARKS

Previous Year Questions

Q1 CBSE 2025

The radius of a cylinder is increasing at the rate of 3 cm/s, and its height is decreasing at the rate of 5 cm/s. Find the rate of change of its volume when radius is 4 cm and height is 7 cm.

Solution: $V = \pi r^2 h$, $\frac{dV}{dt} = \pi(2rh \cdot \frac{dr}{dt} + r^2 \cdot \frac{dh}{dt})$.

At $r=4$, $h=7$, $\frac{dr}{dt}=3$, $\frac{dh}{dt}=-5$:

$$\frac{dV}{dt} = \pi[2(4)(7)(3) + 16(-5)] = \pi[168 - 80] = 88\pi \text{ cm}^3/\text{s}.$$

Q2 CBSE 2025

The radius of a cylinder is decreasing at the rate of 2 cm/s and the altitude is increasing at the rate of 3 cm/s. Find the rate of change of the volume of the cylinder when its radius is 4 cm and altitude is 6 cm.

Solution: $V = \pi r^2 h$, $\frac{dV}{dt} = \pi(2rh \cdot \frac{dr}{dt} + r^2 \cdot \frac{dh}{dt})$.

At $r=4$, $h=6$, $\frac{dr}{dt}=-2$, $\frac{dh}{dt}=3$:

$$\frac{dV}{dt} = \pi[2(4)(6)(-2) + 16(3)] = \pi[-96 + 48] = -48\pi \text{ cm}^3/\text{s}$$

(the volume is decreasing).

Q3 CBSE 2025

Find the intervals in which the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is

(i) strictly increasing (ii) strictly decreasing.

Solution:

$$f'(x) = 12x^3 - 12x^2 - 24x = 12x(x^2 - x - 2) = 12x(x-2)(x+1).$$

The critical points are $x = -1, 0, 2$. Testing the sign of f' in each interval: negative on $(-\infty, -1)$, positive on $(-1, 0)$, negative on $(0, 2)$, positive on $(2, \infty)$. So f is strictly increasing on $(-1, 0) \cup (2, \infty)$, and strictly decreasing on $(-\infty, -1) \cup (0, 2)$.

Q4 CBSE 2023

Find the intervals in which the function f given by $f(x) = -2x^3 - 9x^2 - 12x + 1$ is

(i) strictly increasing (ii) strictly decreasing.

Solution:

$$f'(x) = -6x^2 - 18x - 12 = -6(x^2 + 3x + 2) = -6(x+1)(x+2).$$

The critical points are $x = -2, -1$. Testing signs: $f' < 0$ on $(-\infty, -2)$, $f' > 0$ on $(-2, -1)$, $f' < 0$ on $(-1, \infty)$. So f is strictly increasing on $(-2, -1)$, and strictly decreasing on $(-\infty, -2) \cup (-1, \infty)$.

Q5 CBSE 2023

If the circumference of a circle is increasing at a constant rate, prove that the rate of change of the area of the circle is directly proportional to its radius.

Solution: Let $\frac{dC}{dt} = k$ (a constant). Since $C = 2\pi r$, $\frac{dC}{dt} = 2\pi \cdot \frac{dr}{dt}$, so $\frac{dr}{dt} = \frac{k}{2\pi}$ — also a constant.

Now $A = \pi r^2$, so $\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt} = 2\pi r \cdot \frac{k}{2\pi} = kr$.

Since k is a constant, this shows $\frac{dA}{dt}$ is directly proportional to r .

Q6 CBSE 2023

Consider the statement: "There exists at least one value of $b \in \mathbb{R}$ for which $f(x) = \frac{b}{x}$ ($b \neq 0$) is strictly increasing on $\mathbb{R} - \{0\}$." State True or False. Justify.

Solution: True. $f'(x) = -\frac{b}{x^2}$. Choosing any $b < 0$

(e.g. $b = -1$) makes $-b > 0$, so $f'(x) = \frac{-b}{x^2} > 0$ for every $x \neq 0$ (since $x^2 > 0$ always). So f is strictly increasing on $\mathbb{R} - \{0\}$ for any negative value of b — confirming such a b exists.

Q7 CBSE 2024

The surface area of a cube increases at the rate of 72 cm²/sec. Find the rate of change of its volume when the edge of the cube measures 3 cm.

Solution: $S = 6x^2$, so $\frac{dS}{dt} = 12x \cdot \frac{dx}{dt}$. At $x = 3$: $72 = 12(3) \cdot \frac{dx}{dt}$, so $\frac{dx}{dt} = 2$ cm/s.

Now $V = x^3$, $\frac{dV}{dt} = 3x^2 \cdot \frac{dx}{dt} = 3(9)(2) = 54$ cm³/s.

Q18 CBSE 2025

$\int \frac{3 \cos \sqrt{x}}{\sqrt{x}} dx$ is equal to:

- (A) $-6 \sin \sqrt{x} + C$ (B) $-6 \cos \sqrt{x} + C$
 (C) $6 \cos \sqrt{x} + C$ (D) $6 \sin \sqrt{x} + C$

✓ **Correct Answer:** (D) $6 \sin \sqrt{x} + C$

Detailed Explanation: Put $t = \sqrt{x}$, so $dt = \frac{dx}{2\sqrt{x}}$,

i.e. $\frac{dx}{\sqrt{x}} = 2 dt$.

Then $\int 3 \cos t \cdot 2 dt = 6 \sin t + C = 6 \sin \sqrt{x} + C$.

Q19 CBSE 2025, 2023

$\int \frac{\cot \sqrt{x}}{\sqrt{x}} dx$ is equal to:

- (A) $\log|\sin \sqrt{x}| + C$ (B) $\frac{1}{2} \log|\sin \sqrt{x}| + C$
 (C) $2 \log|\sin \sqrt{x}| + C$ (D) $\frac{\log|\sin \sqrt{x}|}{\sqrt{x}} + C$

✓ **Correct Answer:** (C) $2 \log|\sin \sqrt{x}| + C$

Detailed Explanation: Put $t = \sqrt{x}$, so $\frac{dx}{\sqrt{x}} = 2 dt$.

Then $\int \cot t \cdot 2 dt = 2 \log|\sin t| + C =$
 $2 \log|\sin \sqrt{x}| + C$.

Q20 CBSE 2025, 2024

$\int \frac{\cos 2x}{\sin^2 x \cos^2 x} dx$ is equal to:

- (A) $\cot x + \tan x + C$ (B) $-\cot x + \tan x + C$
 (C) $\cot x - \tan x + C$ (D) $-\cot x - \tan x + C$

✓ **Correct Answer:** (D) $-\cot x - \tan x + C$

Detailed Explanation: $\cos 2x = \cos^2 x - \sin^2 x$,
 so $\frac{\cos 2x}{\sin^2 x \cos^2 x} = \frac{1}{\sin^2 x} - \frac{1}{\cos^2 x} = \operatorname{cosec}^2 x - \sec^2 x$.
 Integrating, $-\cot x - \tan x + C$.

Q21 CBSE 2024

$\int \frac{dx}{x(\log x)^2}$ is equal to:

- (A) $2 \log(\log x) + C$ (B) $-\frac{1}{\log x} + C$
 (C) $\frac{(\log x)^3}{3} + C$ (D) $3(\log x)^3 + C$

✓ **Correct Answer:** (B) $-\frac{1}{\log x} + C$

Detailed Explanation: Put $t = \log x$, so $dt = \frac{dx}{x}$.
 Then $\int \frac{dt}{t^2} = -\frac{1}{t} + C = -\frac{1}{\log x} + C$.

Q22 CBSE 2024

$\int e^x \cdot \frac{x-3}{(x-1)^3} dx$ is equal to:

- (A) $\frac{2e^x}{(x-1)^3} + C$ (B) $-\frac{2e^x}{(x-1)^2} + C$ (C)
 $\frac{e^x}{x-1} + C$ (D) $\frac{e^x}{(x-1)^2} + C$

✓ **Correct Answer:** (D) $\frac{e^x}{(x-1)^2} + C$

Detailed Explanation: $\frac{x-3}{(x-1)^3} = \frac{(x-1)-2}{(x-1)^3} = \frac{1}{(x-1)^2} -$
 $\frac{2}{(x-1)^3}$. With $f = \frac{1}{(x-1)^2}$, $f' = -\frac{2}{(x-1)^3}$,
 so the integrand is $e^x[f + f']$ and the integral is
 $\frac{e^x}{(x-1)^2} + C$.

Q23 CBSE 2024

$\int \frac{e^{3 \log x}}{x^4 + 1} dx$ is equal to:

- (A) $\frac{1}{4} \log(x^4 + 1) + C$ (B) $\frac{1}{4} \log \frac{1}{x^4 + 1} + C$
 (C) $\frac{x^3}{x^4 + 1} + C$ (D) $\frac{e^x}{x^4 + 1} + C$

✓ **Correct Answer:** (A) $\frac{1}{4} \log(x^4 + 1) + C$

Detailed Explanation: $e^{3 \log x} = x^3$,
 so the integrand is $\frac{x^3}{x^4 + 1}$.
 Put $t = x^4 + 1$, $dt = 4x^3 dx$: $\frac{1}{4} \int \frac{dt}{t} = \frac{1}{4} \log(x^4 + 1) + C$.

Q24 CBSE 2023

The value of $\int_{-\pi/2}^{\pi/2} \sin^5 x \cos x dx$ is:

- (A) $\frac{64}{3}$ (B) 0 (C) $2 \sin^6 2$ (D) $\frac{\sin^6 2}{6}$

✓ **Correct Answer:** (B) 0

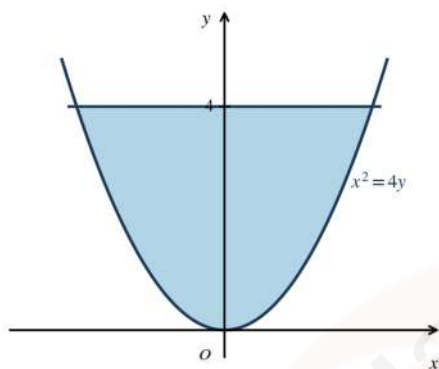
Detailed Explanation: Let $f(x) = \sin^5 x \cos x$.
 Then $f(-x) = (-\sin x)^5 \cos x = -f(x)$,
 so f is odd. For an odd function over $[-a, a]$ the
 integral is 0 (property P_7).

Q15 Board Based

The area bounded by the parabola $x^2 = 4y$ and the line $y = 4$ is:

- (A) $\frac{32}{3}$ sq. units (B) $\frac{16}{3}$ sq. units
(C) 8 sq. units (D) $\frac{64}{3}$ sq. units

✓ **Correct Answer:** (D) $\frac{64}{3}$ sq. units



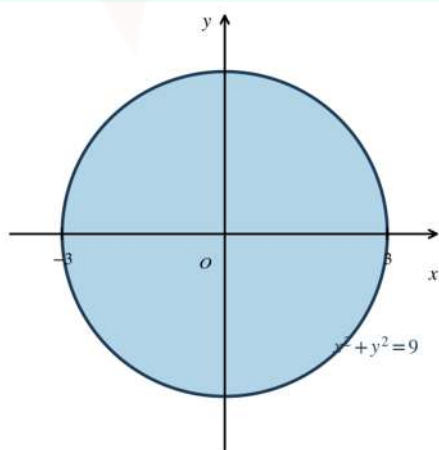
Explanation: $x^2 = 4y$ meets $y = 4$ at $x = \pm 4$.
Horizontal strips, $x = 2\sqrt{y}$ in the first quadrant:
Area $= 2 \int_0^4 2\sqrt{y} dy = 4 \cdot \frac{2}{3} \cdot 8 = \frac{64}{3}$ sq. units.

Q16 Board Based

The area enclosed by the circle $x^2 + y^2 = 9$ is:

- (A) 3π sq. units (B) 6π sq. units
(C) 18π sq. units (D) 9π sq. units

✓ **Correct Answer:** (D) 9π sq. units



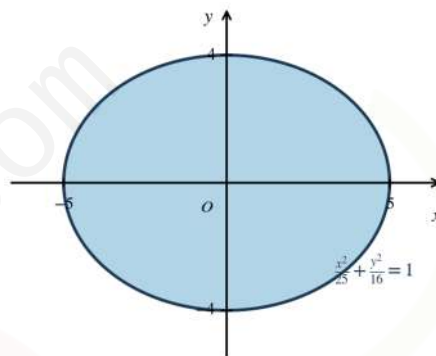
Explanation: Area $= 4 \int_0^3 \sqrt{9 - x^2} dx = 4 \cdot \frac{9}{2} \cdot \frac{\pi}{2} = 9\pi$ sq. units ($= \pi a^2$ with $a = 3$).

Q17 Board Based

The area enclosed by the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ is:

- (A) 20π sq. units (B) 10π sq. units
(C) 40π sq. units (D) 80π sq. units

✓ **Correct Answer:** (A) 20π sq. units



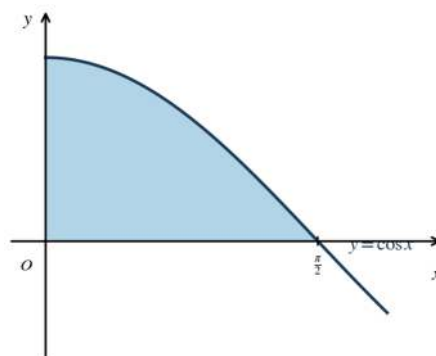
Explanation: Here $a = 5$, $b = 4$. Area $= \pi ab = 20\pi$ sq. units (from $4 \int_0^5 \frac{4}{5} \sqrt{25 - x^2} dx$).

Q18 Board Based

The area under the curve $y = \cos x$ and above the x -axis, from $x = 0$ to $x = \frac{\pi}{2}$, is:

- (A) 1 sq. unit (B) 2 sq. units
(C) $\frac{1}{2}$ sq. unit (D) $\frac{\pi}{2}$ sq. units

✓ **Correct Answer:** (A) 1 sq. unit



Explanation: Area $= \int_0^{\pi/2} \cos x dx = [\sin x]_0^{\pi/2} = 1$ sq. unit.

Q19 Competency Based

The area bounded by the curve $y = \sin x$ and the x -axis between $x = 0$ and $x = 2\pi$ is:

B. Assertion–Reason

★ 1 MARK

Standard CBSE format — choose from options (A)–(D) as defined below each question.

- (A) Both A and R are true and R is the correct explanation of A.
 (B) Both A and R are true but R is not the correct explanation of A.
 (C) A is true but R is false.
 (D) A is false but R is true.

Q1 CBSE 2026

Assertion (A): One of the particular solutions of the differential equation $\frac{dy}{dx} = e^{x+y}$ can be $e^x + e^{-y} = -2$.

Reason (R): $e^x + e^{-y} = C$ is the general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$.

✓ **Correct Answer:** (D) A is false but R is true.

Explanation: Separating, $e^{-y} dy = e^x dx$, so $-e^{-y} = e^x + C_1$, i.e. $e^x + e^{-y} = C$: R is **true**. But $e^x > 0$ and $e^{-y} > 0$, so $e^x + e^{-y}$ is always positive and can never equal -2 : the "solution" in A does not exist (it would need $C = -2 < 0$). A is **false**.

Q2 CBSE 2025

Assertion (A): $x^2 dy = (2xy + y^2) dx$ is a homogeneous differential equation.

Reason (R): A differential equation of the form $\frac{dy}{dx} = F\left(\frac{y}{x}\right)$ is a homogeneous differential equation.

✓ **Correct Answer:** (A) Both A and R are true and R is the correct explanation of A.

Explanation: $\frac{dy}{dx} = \frac{2xy + y^2}{x^2} = 2\frac{y}{x} + \left(\frac{y}{x}\right)^2 = F\left(\frac{y}{x}\right)$, so A is true by the definition in R. R is the definition of a homogeneous equation and is true; it explains A.

Q3 Board Based

Assertion (A): The degree of the differential equation

$\frac{dy}{dx} + \sin\left(\frac{dy}{dx}\right) = 0$ is not defined.

Reason (R): The equation is not a polynomial in $\frac{dy}{dx}$.

✓ **Correct Answer:** (A) Both A and R are true and R is the correct explanation of A.

Explanation: Degree is defined only when the equation is a polynomial in its derivatives. Because $\sin\frac{dy}{dx}$ is not a polynomial in $\frac{dy}{dx}$, the degree is not defined. R is true and is exactly the reason for A.

Q4 Competency Based

Assertion (A): The general solution of a second-order differential equation contains two arbitrary constants.

Reason (R): A particular solution of a first-order differential equation contains one arbitrary constant.

✓ **Correct Answer:** (C) A is true but R is false.

Explanation: The general solution of an equation of order n has n arbitrary constants, so A is true. A particular solution has **no** arbitrary constant, so R is false.

Q5 Board Based

Assertion (A): The integrating factor of $x \frac{dy}{dx} + 2y = x^2$ is e^{2x} .

Reason (R): The integrating factor of $\frac{dy}{dx} + Py = Q$ is $e^{\int P dx}$.

✓ **Correct Answer:** (D) A is false but R is true.

Explanation: R is true. But in $x \frac{dy}{dx} + 2y = x^2$ the coefficient of $\frac{dy}{dx}$ is x , so first divide by x : $\frac{dy}{dx} + \frac{2}{x}y = x$. Then $P = \frac{2}{x}$ and the I.F. is $e^{2 \log x} = x^2$. A is false.

Q6 Competency Based

Assertion (A): $\frac{dy}{dx} = \frac{y}{x}$ is a variable separable differential equation.

Reason (R): $\frac{dy}{dx} = \frac{y}{x}$ is a homogeneous differential equation.

C. Very Short Answer

★ 2 MARKS

Q1 CBSE 2026

Three honey bees were found flying along the vectors $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\vec{b} = 4\hat{j} - 2\hat{k}$ and $\vec{c} = 3\hat{i} + 2\hat{k}$ respectively. Find the value of λ such that the path for $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$.

Solution:

$$\vec{a} + \lambda\vec{b} = 2\hat{i} + (4\lambda - 3)\hat{j} + (1 - 2\lambda)\hat{k}.$$

Perpendicular to $\vec{c}$: $(\vec{a} + \lambda\vec{b}) \cdot \vec{c} = 0$.

$$6 + 0 + 2(1 - 2\lambda) = 8 - 4\lambda = 0, \text{ so } \lambda = 2.$$

✓ **Answer:** $\lambda = 2$

Q2 CBSE 2026

If A , B and C are three non-collinear points such that $\vec{AB} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{AC} = 2\hat{i} - 3\hat{j}$, then find the area of ABC .

Solution: $\vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 2 & -3 & 0 \end{vmatrix} = -3\hat{i} - 2\hat{j} - 7\hat{k}$, with magnitude $\sqrt{9 + 4 + 49} = \sqrt{62}$.

$$\text{Area} = \frac{1}{2}\sqrt{62} = \frac{\sqrt{62}}{2} \text{ sq. units.}$$

✓ **Answer:** $\frac{\sqrt{62}}{2}$ sq. units

Q3 CBSE 2026

Find a unit vector perpendicular to both of the vectors $\vec{AB}$ and $\vec{BC}$, where A , B and C are $(3, -1, 2)$, $(1, -1, -3)$ and $(4, -3, 1)$ respectively.

Solution: $\vec{AB} = -2\hat{i} - 5\hat{k}$ and $\vec{BC} = 3\hat{i} - 2\hat{j} + 4\hat{k}$.

$$\vec{AB} \times \vec{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & -5 \\ 3 & -2 & 4 \end{vmatrix} = -10\hat{i} - 7\hat{j} + 4\hat{k},$$

$$\text{with magnitude } \sqrt{100 + 49 + 16} = \sqrt{165}.$$

$$\text{Unit vectors: } \pm \frac{-10\hat{i} - 7\hat{j} + 4\hat{k}}{\sqrt{165}}.$$

✓ **Answer:** $\pm \frac{-10\hat{i} - 7\hat{j} + 4\hat{k}}{\sqrt{165}}$

Q4 CBSE 2026

If for two unit vectors $\vec{a}$ and $\vec{b}$, $|\vec{a} + 2\vec{b}| = |2\vec{a} - \vec{b}|$, then find the angle between $\vec{a}$ and $\vec{b}$.

Solution: Square both sides:

$$|\vec{a}|^2 + 4\vec{a} \cdot \vec{b} + 4|\vec{b}|^2 = 4|\vec{a}|^2 - 4\vec{a} \cdot \vec{b} + |\vec{b}|^2.$$

With unit vectors: $5 + 4\cos\theta = 5 - 4\cos\theta$,

$$\text{so } \cos\theta = 0 \text{ and } \theta = \frac{\pi}{2}.$$

✓ **Answer:** $\frac{\pi}{2}$

Q5 CBSE 2026

In a parallelogram $PQRS$, $\vec{PQ} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{QR} = \hat{i} - 2\hat{j} + \hat{k}$. Find the unit vectors parallel to the diagonals PR and SQ .

Solution: $\vec{PR} = \vec{PQ} + \vec{QR} = 3\hat{i} + \hat{j} + 2\hat{k}$, with magnitude $\sqrt{9 + 1 + 4} = \sqrt{14}$.

In a parallelogram $\vec{PS} = \vec{QR}$, so $\vec{SQ} = \vec{PQ} - \vec{PS} = \vec{PQ} - \vec{QR} = \hat{i} + 5\hat{j}$, with magnitude $\sqrt{26}$.

Unit vectors: $\frac{3\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{14}}$ along PR and $\frac{\hat{i} + 5\hat{j}}{\sqrt{26}}$ along SQ .

✓ **Answer:** Along PR : $\frac{3\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{14}}$; along SQ : $\frac{\hat{i} + 5\hat{j}}{\sqrt{26}}$

Q6 CBSE 2026

If the position vectors of three points A , B and C are $3\hat{i} + \hat{j}$, $5\hat{i} + 6\hat{j} - 3\hat{k}$ and $4\hat{j}$ respectively, then show that they form an isosceles triangle.

Solution: $\vec{AB} = 2\hat{i} + 5\hat{j} - 3\hat{k}$,

$$\text{so } |AB|^2 = 4 + 25 + 9 = 38. \vec{BC} = -5\hat{i} - 2\hat{j} + 3\hat{k},$$

$$\text{so } |BC|^2 = 25 + 4 + 9 = 38. \text{ And } \vec{AC} = -3\hat{i} + 3\hat{j},$$

$$\text{so } |AC|^2 = 18. |AB| = |BC| = \sqrt{38},$$

so the triangle is isosceles.

✓ **Answer:** Shown: $AB = BC = \sqrt{38}$.

Q7 CBSE 2026

Vectors $\vec{a} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{k}$ represent the two adjacent sides of a parallelogram. Find the vectors representing its diagonals and hence find their lengths.

Q50 Board Based

The shortest distance between the lines $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$ and $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ is:

- (A) $\sqrt{29}$ (B) $2\sqrt{29}$ (C) $4\sqrt{29}$ (D) $\frac{2}{\sqrt{29}}$

✓ **Correct Answer:** (B) $2\sqrt{29}$

Explanation: $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 1 \\ 7 & -6 & 1 \end{vmatrix}$
 $= 4\hat{i} + 6\hat{j} + 8\hat{k}$, with magnitude $\sqrt{116} = 2\sqrt{29}$.
 $\vec{a}_2 - \vec{a}_1 = -4\hat{i} - 6\hat{j} - 8\hat{k}$ and its dot product with $\vec{b}_1 \times \vec{b}_2$ is -116 . Distance $= \frac{116}{2\sqrt{29}} = 2\sqrt{29}$.

Q51 Competency Based

The distance between the parallel lines $\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(4\hat{i} - 2\hat{j} + 2\hat{k})$ is:

- (A) $\frac{3\sqrt{2}}{2}$ (B) $\sqrt{6}$ (C) $\sqrt{2}$ (D) $\frac{\sqrt{22}}{2}$

✓ **Correct Answer:** (A) $\frac{3\sqrt{2}}{2}$

Explanation: $\vec{a}_2 - \vec{a}_1 = \hat{i} - 2\hat{j} - \hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$,
 so $(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & -1 \\ 2 & -1 & 1 \end{vmatrix}$
 $= -3\hat{i} - 3\hat{j} + 3\hat{k}$, of magnitude $\sqrt{27} = 3\sqrt{3}$.
 Distance $= \frac{3\sqrt{3}}{\sqrt{6}} = \frac{3\sqrt{2}}{2}$ units.

Q52 Board Based

The foot of the perpendicular drawn from the point (1,2,3) on the line $\frac{x}{1} = \frac{y}{1} = \frac{z}{1}$ is:

- (A) (2, 2, 2) (B) (1, 1, 1) (C) (3, 3, 3) (D) (1, 2, 3)

✓ **Correct Answer:** (A) (2, 2, 2)

Explanation: A general point of the line is $Q(\lambda, \lambda, \lambda)$ and the direction ratios of the line are (1, 1, 1).
 $\vec{PQ} = (-1 + \lambda, -2 + \lambda, -3 + \lambda)$. Since $\vec{PQ} \perp$ line:
 $(-1 + \lambda) + (-2 + \lambda) + (-3 + \lambda) = 0$, i.e. $-6 + 3\lambda = 0$, so $\lambda = 2$. Foot of the perpendicular:
 $Q(2, 2, 2)$.

Q53 Competency Based

The image of the point (1,2,3) in the x -axis is:

- (A) (-1, 2, 3) (B) (1, -2, -3)
 (C) (1, -2, 3) (D) (-1, -2, -3)

✓ **Correct Answer:** (B) (1, -2, -3)

Explanation: The foot of the perpendicular from (1,2,3) on the x -axis is (1,0,0). The foot is the mid-point of the point and its image, so the image is $2(1,0,0) - (1,2,3) = (1, -2, -3)$.

Q54 Board Based

A line through the origin is perpendicular to the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x}{2} = \frac{y}{0} = \frac{z}{-1}$. Its direction ratios are:

- (A) 2, -7, -4 (B) -2, 7, -4
 (C) 1, 2, 3 (D) -2, -7, 4

✓ **Correct Answer:** (B) -2, 7, -4

Explanation: Direction $= \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 0 & -1 \end{vmatrix} =$
 $-2\hat{i} + 7\hat{j} - 4\hat{k}$. So the direction ratios are $\langle -2, 7, -4 \rangle$ (or any non-zero multiple).
 Check: $-2 + 14 - 12 = 0$ and $-4 + 0 + 4 = 0$.

Q55 Competency Based

If α, β, γ are the angles which a line makes with the positive directions of the coordinate axes, then $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$ equals:

- (A) 1 (B) 0 (C) -3 (D) -1

✓ **Correct Answer:** (D) -1

Explanation: $\cos 2\theta = 2\cos^2 \theta - 1$, so the sum is $2(\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma) - 3$. Since $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$, the sum is $2 - 3 = -1$.

Q56 Competency Based

The lines $\vec{r} = \hat{i} + 2\hat{j} + \hat{k} + \lambda(\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = \hat{i} + \hat{j} + \hat{k} + \mu(\hat{i} - \hat{j} + 2\hat{k})$ are:

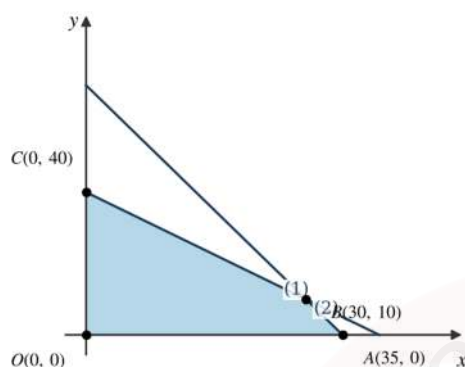
- (A) parallel lines (B) intersecting lines
 (C) coincident lines (D) skew lines

C. Very Short Answer

★ 2 MARKS

Q1 CBSE 2026

Draw the feasible region of the constraints $2x + y \leq 70$, $x + y \leq 40$ and $x \geq 0, y \geq 0$, and write its corner points.



Feasible region; lines: $2x + y = 70$; $x + y = 40$

Solution:

Graph. Line $2x + y = 70$ meets the axes at $(35, 0)$ and $(0, 70)$. Line $x + y = 40$ meets the axes at $(40, 0)$ and $(0, 40)$.

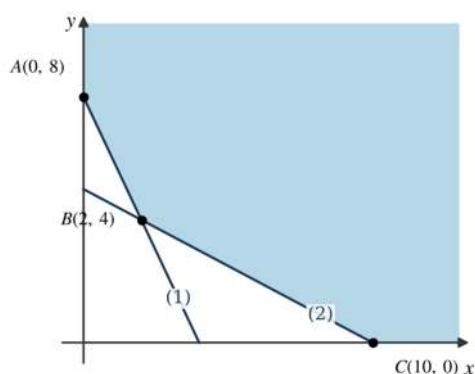
Test $O(0, 0)$ in each inequality and shade the side that satisfies it, together with $x \geq 0, y \geq 0$.

The corner points are $O(0, 0)$, $A(35, 0)$, $B(30, 10)$, $C(0, 40)$.

✓ **Answer:** Corner points: $(0, 0)$, $(35, 0)$, $(30, 10)$, $(0, 40)$.

Q2 CBSE 2026

Draw the feasible region of the constraints $2x + y \geq 8$, $x + 2y \geq 10$ and $x \geq 0, y \geq 0$, and write its corner points.



Feasible region; lines: $2x + y = 8$; $x + 2y = 10$

Solution:

Graph. Line $2x + y = 8$ meets the axes at $(4, 0)$ and $(0, 8)$. Line $x + 2y = 10$ meets the axes at $(10, 0)$ and $(0, 5)$.

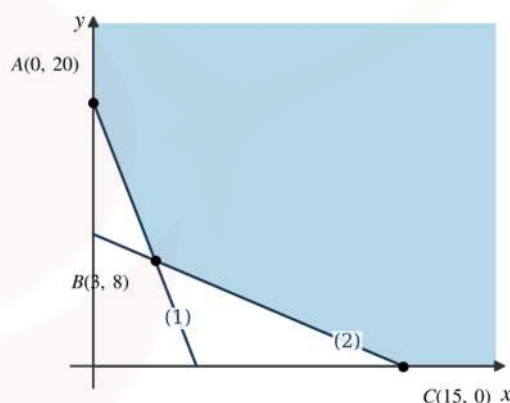
Test $O(0, 0)$ in each inequality and shade the side that satisfies it, together with $x \geq 0, y \geq 0$.

The corner points are $A(0, 8)$, $B(2, 4)$, $C(10, 0)$. The region is unbounded.

✓ **Answer:** Corner points: $(0, 8)$, $(2, 4)$, $(10, 0)$.

Q3 CBSE 2026

Draw the feasible region of the constraints $4x + y \geq 20$, $2x + 3y \geq 30$ and $x \geq 0, y \geq 0$, and write its corner points.



Feasible region; lines: $4x + y = 20$; $2x + 3y = 30$

Solution:

Graph. Line $4x + y = 20$ meets the axes at $(5, 0)$ and $(0, 20)$. Line $2x + 3y = 30$ meets the axes at $(15, 0)$ and $(0, 10)$.

Test $O(0, 0)$ in each inequality and shade the side that satisfies it, together with $x \geq 0, y \geq 0$.

The corner points are $A(0, 20)$, $B(3, 8)$, $C(15, 0)$. The region is unbounded.

✓ **Answer:** Corner points: $(0, 20)$, $(3, 8)$, $(15, 0)$.

Q4 CBSE 2025

The corner points of the feasible region determined by a system of linear inequalities are $A(25, 0)$, $B(16, 16)$ and $C(0, 24)$. Find the maximum and the minimum value of $Z = 4x + 3y$, and the points at which they occur.

B. Assertion–Reason

★ 1 MARK

Standard CBSE format — choose from options (A)–(D) as defined below each question.

- (A) Both A and R are true and R is the correct explanation of A.
 (B) Both A and R are true but R is not the correct explanation of A.
 (C) A is true but R is false.
 (D) A is false but R is true.

Q1 CBSE 2026

Assertion (A): A die is thrown. If B is the event "the number is odd" and A is the event "the number is prime", then $P(A|B) = \frac{2}{3}$.

Reason (R): For events A and B , $P(A|B) = \frac{P(A \cap B)}{P(B)}$.

✓ **Correct Answer:** (C) A is true but R is false.

Explanation: Odd numbers: 1, 3, 5; primes among them: 3, 5. So $P(A|B) = \frac{2}{3}$: A is true. The correct formula is $P(A|B) = \frac{P(A \cap B)}{P(B)}$ (intersection, not union): R is false.

Q2 CBSE 2026

Assertion (A): If A and B are independent events with $P(B) \neq 0$, then $P(A|B) = P(A)$.

Reason (R): For independent events A and B , $P(A \cap B) = P(A) \cdot P(B)$.

✓ **Correct Answer:** (A) Both A and R are true and R is the correct explanation of A.

Explanation: $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)P(B)}{P(B)} = P(A)$. A is true, and R is exactly the fact used to prove it, so R explains A.

Q3 CBSE 2025

Assertion (A): If A and B are mutually exclusive events with $P(A) > 0$ and $P(B) > 0$, then A and B are independent.

Reason (R): Two events A and B are independent if and only if $P(A \cap B) = P(A)P(B)$.

✓ **Correct Answer:** (D) A is false but R is true.

Explanation: Mutually exclusive means $P(A \cap B) = 0$, but $P(A)P(B) > 0$, so the product rule fails: A is false. R is the definition of independence: R is true.

Q4 CBSE 2024

Assertion (A): If R and S are events with $P(S) > 0$ and $P(R|S) = 1$, then SR (apart from events of probability 0).

Reason (R): If $P(A \cap B) = P(B)$, then BA (apart from events of probability 0).

✓ **Correct Answer:** (A) Both A and R are true and R is the correct explanation of A.

Explanation: $P(R|S) = 1$ means $P(R \cap S) = P(S)$: every outcome of S lies in R . R states this general fact, so R explains A. Both are true.

Q5 CBSE 2023

Assertion (A): Two fair coins are tossed. The probability of getting two heads, given that at least one head has appeared, is $\frac{1}{3}$.

Reason (R): For events E and F with $P(E) > 0$, $P(F|E) = \frac{P(E \cap F)}{P(E)}$.

✓ **Correct Answer:** (A) Both A and R are true and R is the correct explanation of A.

Explanation: Given at least one head, the reduced sample space is $\{HH, HT, TH\}$ (3 outcomes); only HH is favourable, so the probability is $\frac{1}{3}$. A is true. R is the formula used, so R explains A.

Q6 Board Based

Assertion (A): A fair coin is tossed twice. The probability that both tosses show heads is $\frac{1}{4}$.

Reason (R): For any two events A and B , $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

✓ **Correct Answer:** (B) Both A and R are true but R is not the correct explanation of A.

Explanation: The two tosses are independent: $P(HH) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$, so A is true. R is also a true