Observations of Math Lab Manual Activities for Class 12

1. To verify that the relation R in the set L of all lines in a plane, defined by R = {(l, m) : l ⊥ m} is symmetric but neither reflexive nor transitive.

Observations:

1. In Fig. 1, no line is perpendicular to itself, so the relation R = {(l, m) : l ⊥ m} ___**is not**___ reflexive (is/is not).

2. In Fig. 1, $l_1 \perp l_2$. Is $l_2 \perp l_1$? __**Yes**____ (Yes/No)

∴ $(l_1, l_2) \in R \Rightarrow (l_2, l_1)$ ___∈____ R (∉/∈)

Similarly, $l_3 \perp l_1$. Is $l_1 \perp l_3$? ___**Yes**___ (Yes/No)

∴ $(l_3, l_1) \in R \Rightarrow (l_1, l_3)$ ___∈____ R (∉/∈)

Also, $l_6 \perp l_7$. Is $l_7 \perp l_6$? ___**Yes**___ (Yes/No)

∴ $(l_6, l_7) \in R \Rightarrow (l_7, l_6)$ ___∈____ R (∉/∈)

∴ The relation R …**is**… symmetric (is/is not)

3. In Fig. 1, $l_2 \perp l_1$ and $l_1 \perp l_3$. Is $l_2 \perp l_3$? __**No**____ (Yes/No)

∴ $(l_2, l_1) \in R$ and $(l_1, l_3) \in R \Rightarrow (l_2, l_1)$ ___∉____ R (∉/∈)

∴ The relation R …**is not**… transitive (is/is not)

2. To explore the principal value of the function $\sin^{-1} x$ using a unit circle.

Observations:

1. sine function is non-negative in ____**first**_____ and _____**second**_____ quadrants.

2. For the quadrants 3rd and 4th, sine function is ____**negative**_____.

3. θ = arc sin y ⇒ y = ___**sin**_____ θ where $-\frac{\pi}{2} \le \theta \le$ ___$\frac{\pi}{2}$_____.

4. The other domains of sine function on which it is one-one and onto provides ___**range**______ for arc sine function.

3. To verify that for a function f to be continuous at given point x_0, $\Delta y = |f(x_o + \Delta x) - f(x_o)|$ is arbitrarily small provided. Δx is sufficiently small.

Observations:

S. No.	Value of increment in x_o	Corresponding increment in y
1.	$|\Delta x_1| =$ ___**9**____	$|\Delta y_1| =$ ___**3**____
2.	$|\Delta x_2| =$ ___**8**____	$|\Delta y_2| =$ ___**2.8**____
3.	$|\Delta x_3| =$ ___**7**____	$|\Delta y_3| =$ ___**2.6**____
4.	$|\Delta x_4| =$ ___**6**____	$|\Delta y_4| =$ ___**2.3**____
5.	$|\Delta x_5| =$ ___**5**____	$|\Delta y_5| =$ ___**2**____
6.	$|\Delta x_6| =$ ___**4**____	$|\Delta y_6| =$ ___**1.7**____
7.	$|\Delta x_7| =$ ___**3**____	$|\Delta y_7| =$ ___**1.4**____
8.	$|\Delta x_8| =$ ___**2**____	$|\Delta y_8| =$ ___**1.1**____
9.	$|\Delta x_9| =$ ___**1**____	$|\Delta y_9| =$ ___**0.8**____

2. So, Δy becomes ____**smaller**_____ when Δx becomes smaller.

3. Thus $\lim_{\Delta x \to 0} \Delta y = 0$ for a continuous function.

4. To understand the concepts of local maxima, local minima and point of inflexion.

Observations:

1. Sign of the slope of the tangent (first derivative) at a point on the curve to the immediate left of A is ___**negative**____.

2. Sign of the slope of the tangent (first derivative) at a point on the curve to the immediate right of A is___**positive**____.

3. Sign of the first derivative at a point on the curve to immediate left of B is ___ **negative** ____.

4. Sign of the first derivative at a point on the curve to immediate right of B is ___ **positive** ____.

5. Sign of the first derivative at a point on the curve to immediate left of C is ___ **positive** ____.

6. Sign of the first derivative at a point on the curve to immediate right of C is ___ **negative** ____.

7. Sign of the first derivative at a point on the curve to immediate left of D is ___ **positive** ____.

8. Sign of the first derivative at a point on the curve to immediate right of D is ___ **negative** ____.

9. Sign of the first derivative at a point immediate left of P is ___ **positive** ____ and immediate right of P is___ **positive** ____.

10. A and B are points of local ___**minima**____.

11. C and D are points of local ___**maxima**____.

12. P is a point of ___**inflexion**____.

5. To construct an open box of maximum volume from a given rectangular sheet by cutting equal squares from each corner.

Observations:

1. V_1 = Volume of the open box (when x = 1.6) = ...**182.78**..........

2. V_2 = Volume of the open box (when x = 1.9) = ...**190.83**..........

3. V = Volume of the open box (when x = 2.1) = ...**192.44**..........

4. V_3 = Volume of the open box (when x = 2.2) = ...**192.19**..........

5. V_4 = Volume of the open box (when x = 2.4) = ...**189.69**..........

6. V_5 = Volume of the open box (when x = 3.2) = ...**156.67**..........

7. Volume V_1 is _____**<**_______ than volume V.

8. Volume V_2 is _____**<**_______ than volume V.

9. Volume V_3 is _____**>**_______ than volume V.

10. Volume V_4 is ____**>**________ than volume V.

11. Volume V_5 is ____**<**________ than volume V.

So, Volume of the open box is maximum when x = ___**2.1**_____.

6. To verify that amongst all the rest angles of the same perimeter, the square has the maximum area.

Observations:

1. Perimeter of each rectangle R_1, R_2, R_3, R_4, R_5, R_6, R_7 is ___**48 cm**______.

2. Area of the rectangle R_3 ___**<**_____ than the area of rectangle R_5.

3. Area of the rectangle R_6 ___**<**____ than the area of rectangle R_5.

4. The rectangle R_5 has the dimensions __**12 cm**__ × __**12 cm**__ and hence it is a ___**square**___.

5. Of all the rectangles with same perimeter, the ___**square**___ has the maximum area.

7. To verify geometrically that $\vec{c} \times (\vec{a} + \vec{b}) = \vec{c} \times \vec{a} + \vec{c} \times \vec{b}$.

Observations:

$|\vec{c}| = |\overrightarrow{OA}| = OA =$ ____**6 cm**_____

$|\vec{a} + \vec{b}| = |\overrightarrow{OC}| = OC =$ __**7 cm**____

CL = ___**4.9**____

$|\vec{c} \times (\vec{a} + \vec{b})|$ = Area of parallelogram OAPC = (OA) (CL) = ____**29.4**_____ sq. units … (i)

$|\vec{c} \times \vec{a}|$ = Area of parallelogram OAQB = (OA) (BM) = __**6**__ x __**3.4**___ = ___**20.4**___ … (ii)

$|\vec{c} \times \vec{b}|$ = Area of parallelogram BQPC = (OA) (CN) = __**6**__ x __**1.5**___ = ___**9**___ … (iii)

From (i), (ii) and (iii),

Area of parallelogram OAPC = Area of parallelgram OAQB + Area of Parallelgram = ___**29.4**_____.

Thus $|\vec{c} \times (\vec{a} + \vec{b})| = |\vec{c} \times \vec{a}| + |\vec{c} \times \vec{b}|$

$\vec{c} \times \vec{a}, \vec{c} \times \vec{b}$ and $\vec{c} \times (\vec{a} + \vec{b})$ are all in the direction of ___**perpendicular**___ to the plane of paper.

Therefore, $\vec{c} \times (\vec{a} + \vec{b}) = \vec{c} \times \vec{a} +$ __$\vec{c} \times \vec{b}$__.

8. To verify that angle in a semi-circle is a right angle, using vector method.

Observations:

By actual measurement.

$|\overrightarrow{OP}| = |\overrightarrow{OA}| = |\overrightarrow{OB}| = |\overrightarrow{OQ}| = r = a = p =$ __**10 cm**____,

$|\overrightarrow{AP}| =$ ___$\mathbf{10\sqrt{2}}$___, $|\overrightarrow{BP}| =$ __ $\mathbf{10\sqrt{2}}$___ , $|\overrightarrow{AB}| =$____**20 cm**_______.

$|\overrightarrow{AQ}| =$ __$|\vec{\boldsymbol{r}} + \vec{\boldsymbol{a}}|$__, $|\overrightarrow{BQ}| =$ ___$|\vec{\boldsymbol{r}} - \vec{\boldsymbol{a}}|$___

$|\overrightarrow{AP}|^2 + |\overrightarrow{BP}|^2 =$ __$|\overrightarrow{\boldsymbol{AB}}|^{\mathbf{2}}$__, $|\overrightarrow{AQ}|^2 + |\overrightarrow{BQ}|^2 =$ ___$|\overrightarrow{\boldsymbol{AB}}|^{\mathbf{2}}$___

So, $\angle APB =$ __$\mathbf{90^o}$__ and $\overrightarrow{AP}.\overrightarrow{BP}$ = ___**0**____, $\angle AQB =$ ___$\mathbf{90^o}$___ and $\overrightarrow{AQ}.\overrightarrow{BQ} =$ __**0**__.

Similarly, for points R, S, T, __**concyclic points**__

∠ARB = ___$\mathbf{90^o}$___, ∠ASB = ___$\mathbf{90^o}$___, ∠ATB = ___$\mathbf{90^o}$___.

i.e., angle in a semi-circle is a right angle.

9. To measure the shortest distance between two skew lines and verify it analytically.

Observations:

1. Coordinates of point P are ___**(2,2,0)**____.
2. Coordinates of point Q are ____**(7,6,0)**____.
3. Coordinates of point R are ____**(1,6,2)**____.
4. Coordinates of point S are ____**(6,2,4)**____.
5. Equation of line PQ is ___ $\mathbf{\frac{x-2}{5} = \frac{y-2}{4} = \frac{z-0}{0}}$ _____.
6. Equation of line RS is ___ $\mathbf{\frac{x-1}{5} = \frac{y-6}{-4} = \frac{z-2}{2}}$ _____.

Shortest distance between PQ and RS analytically = ___**2.6 cm**___.

Shortest distance by actual measurement = ____**2.59 cm** ____.

The results so obtained are ___ **equal (approximately)**___.

10. To explain the computation of the conditional probability of a given event A, when event B has already occurred, through an example of throwing a pair of dice.

Observations:

1. Outcome(s) favourable to A : ___**(4,4)**_____, n (A) = ___**1**______.

2. Outcomes favourable to B : _**(1,4), (2,4), (3,4), (4,4), (5,4), (6,4), (4,1), (4,2), (4,3), (4,5), (4,6)**___, n (B) = ___**11**____.

3. Outcomes favourable to A ∩ B : ____**(4,4)**____, n (A ∩ B) = ____**1**_____.

4. P (A ∩ B) = ____ $\frac{1}{36}$ _____.

5. P (A | B) = ___ $\frac{P(A\cap B)}{P(B)}$ ___ = ___ $\frac{1}{11}$ __.