Activity 1

Objective

To verify that the relation R in the set L of all lines in a plane, defined by R = {$(l, m) : l \perp m$} is symmetric but neither reflexive nor transitive.

Material Required

A piece of plywood, some pieces of wires (8), nails, white paper, glue etc.

Method of Construction

Take a piece of plywood and paste a white paper on it. Fix the wires randomly on the plywood with the help of nails such that some of them are parallel, some are perpendicular to each other and some are inclined as shown in Fig.1.

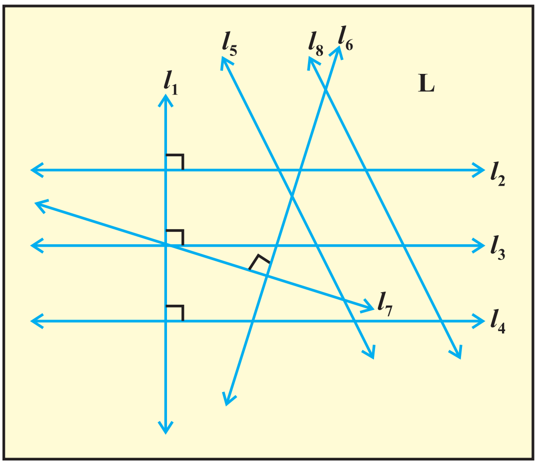

Fig. 1

Demonstration

1. Let the wires represent the lines $l_1, l_2, ..., l_8$.
2. l_1 is perpendicular to each of the lines l_2, l_3, l_4. [see Fig. 1]

3. l_6 is perpendicular to l_7.

4. l_2 is parallel to l_3, l_3 is parallel to l_4 and l_5 is parallel to l_8.

5. $(l_1, l_2), (l_1, l_3), (l_1, l_4), (l_6, l_7) \in$ R

Observation

1. In Fig. 1, no line is perpendicular to itself, so the relation R = $\{(l, m) : l \perp m\}$ ______ reflexive (is/is not).

2. In Fig. 1, $l_1 \perp l_2$. Is $l_2 \perp l_1$?______ (Yes/No)

   $\therefore$ $(l_1, l_2) \in R \Rightarrow (l_2, l_1)$ ______ R $(\notin / \in)$

   Similarly, $l_3 \perp l_1$. Is $l_1 \perp l_3$? _______ (Yes/No)

   $\therefore$ $(l_3, l_1) \in R \Rightarrow (l_1, l_3)$ ______ R $(\notin / \in)$

   Also, $l_6 \perp l_7$. Is $l_7 \perp l_6$? _______ (Yes/No)

   $\therefore$ $(l_6, l_7) \in R \Rightarrow (l_7, l_6)$ ______ R $(\notin / \in)$

   $\therefore$ The relation R .... symmetric (is/is not)

3. In Fig. 1, $l_2 \perp l_1$ and $l_1 \perp l_3$. Is $l_2 \perp l_3$? ... (Yes/No)

   i.e., $(l_2, l_1) \in R$ and $(l_1, l_3) \in R \Rightarrow (l_2, l_3)$ ______ R $(\notin / \in)$

   $\therefore$ The relation R .... transitive (is/is not).

Application

This activity can be used to check whether a given relation is an equivalence relation or not.

Note

1. In this case, the relation is not an equivalence relation.
2. The activity can be repeated by taking some more wire in different positions.

Activity 6

Objective

To explore the principal value of the function $\sin^{-1}x$ using a unit circle.

Material Required

Cardboard, white chart paper, rails, ruler, adhesive, steel wires and needle.

Method of Construction

1. Take a cardboard of a convenient size and paste a white chart paper on it.
2. Draw a unit circle with centre O on it.
3. Through the centre of the circle, draw two perpendicular lines X′OX and YOY′ representing x-axis and y-axis, respectively as shown in Fig. 6.1.
4. Mark the points A, C, B and D, where the circle cuts the x-axis and y-axis, respectively as shown in Fig. 6.1.
5. Fix two rails on opposite sides of the cardboard which are parallel to y-axis. Fix one steel wire between the rails such that the wire can be moved parallel to x-axis as shown in Fig. 6.2.

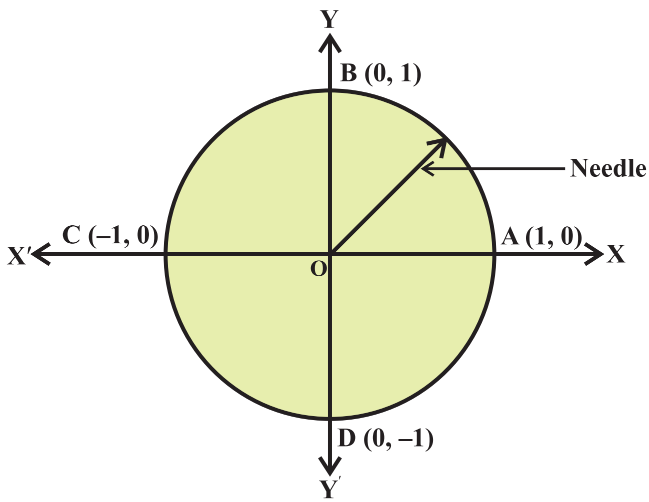

Fig. 6.1

6. Take a needle of unit length. Fix one end of it at the centre of the circle and the other end to move freely along the circle Fig. 6.2.

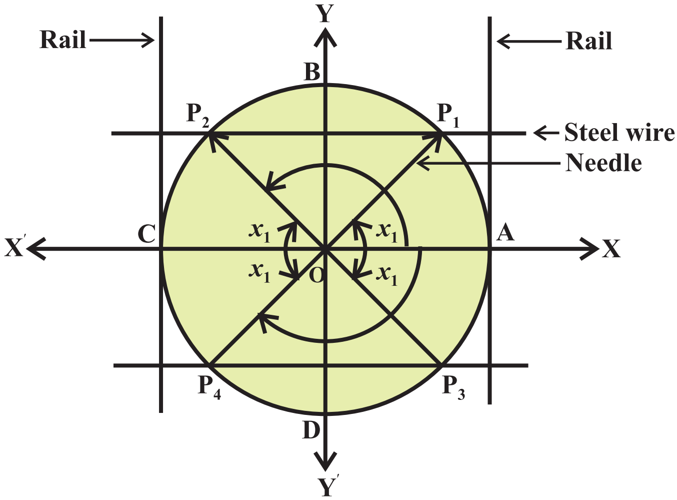

Fig. 6.2

DEMONSTRATION

1. Keep the needle at an arbitrary angle, say x_1 with the positive direction of x-axis. Measure of angle in radian is equal to the length of intercepted arc of the unit circle.

2. Slide the steel wire between the rails, parallel to x-axis such that the wire meets with free end of the needle (say P_1) (Fig. 6.2).

3. Denote the y-coordinate of the point P_1 as y_1, where y_1 is the perpendicular distance of steel wire from the x-axis of the unit circle giving $y_1 = \sin x_1$.

4. Rotate the needle further anticlockwise and keep it at the angle $\pi - x_1$. Find the value of y-coordinate of intersecting point P_2 with the help of sliding steel wire. Value of y-coordinate for the points P_1 and P_2 are same for the different value of angles, $y_1 = \sin x_1$ and $y_1 = \sin (\pi - x_1)$. This demonstrates that sine function is not one-to-one for angles considered in first and second quadrants.

5. Keep the needle at angles $-x_1$ and $(-\pi + x_1)$, respectively. By sliding down the steel wire parallel to x-axis, demonstrate that y-coordinate for the points P_3 and P_4 are the same and thus sine function is not one-to-one for points considered in 3rd and 4th quadrants as shown in Fig. 6.2.

6. However, the y-coordinate of the points P_3 and P_1 are different. Move the needle in anticlockwise direction starting from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$ and look at the behaviour of y-coordinates of points P_5, P_6, P_7 and P_8 by sliding the steel wire parallel to x-axis accordingly. y-coordinate of points P_5, P_6, P_7 and P_8 are different (see Fig. 6.3). Hence, sine function is one-to-one in the domian $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and its range lies between – 1 and 1.

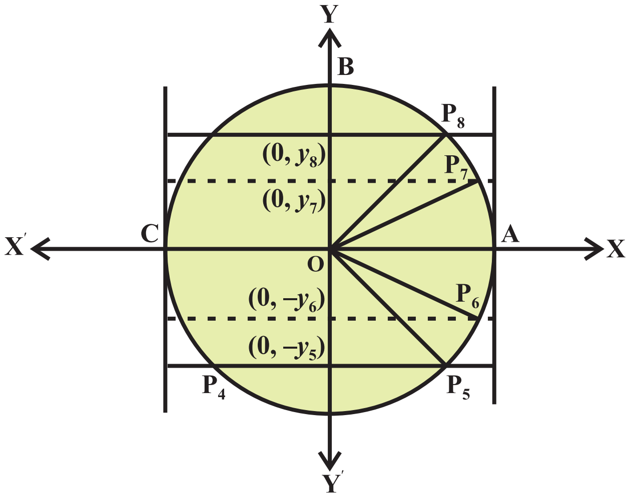

Fig. 6.3

7. Keep the needle at any arbitrary angle say θ lying in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and denote the y-coordinate of the intersecting point P_9 as y. (see Fig. 6.4). Then $y = \sin\theta$ or θ = arc $\sin^{-1}y$) as sine function is one-one and onto in the domain $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and range [–1, 1]. So, its inverse arc sine function exist. The domain of arc sine function is [–1, 1] and

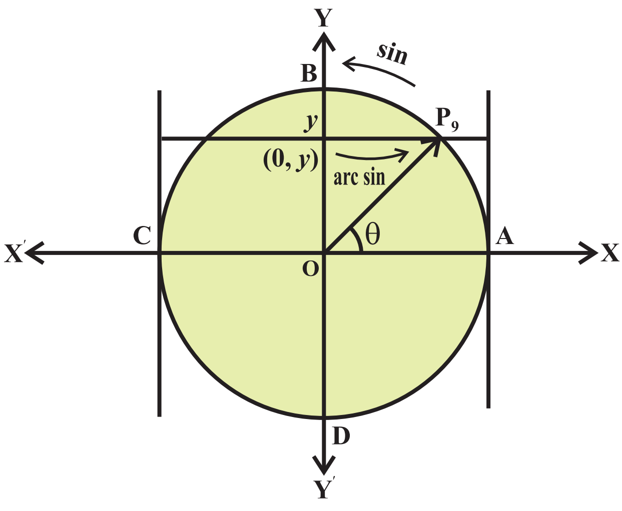

Fig. 6.4

range is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. This range is called the principal value of arc sine function (or $\sin^{-1}$ function).

Observation

1. sine function is non-negative in _________ and __________ quadrants.
2. For the quadrants 3rd and 4th, sine function is _________.
3. θ = arc sin $y \Rightarrow y$ = ________ θ where $-\frac{\pi}{2} \le \theta \le$ ________.
4. The other domains of sine function on which it is one-one and onto provides _________ for arc sine function.

Application

This activity can be used for finding the principal value of arc cosine function ($\cos^{-1}y$).

Activity 10

Objective

To verify that for a function f to be continuous at given point x_0,

$\Delta y = \left| f(x_0 + \Delta x) - f(x_0) \right|$ is

arbitrarily small provided. Δx is sufficiently small.

Material Required

Hardboard, white sheets, pencil, scale, calculator, adhesive.

Method of Construction

1. Paste a white sheet on the hardboard.
2. Draw the curve of the given continuous function as represented in the Fig. 10.
3. Take any point A (x_0, 0) on the positive side of x-axis and corresponding to this point, mark the point P (x_0, y_0) on the curve.

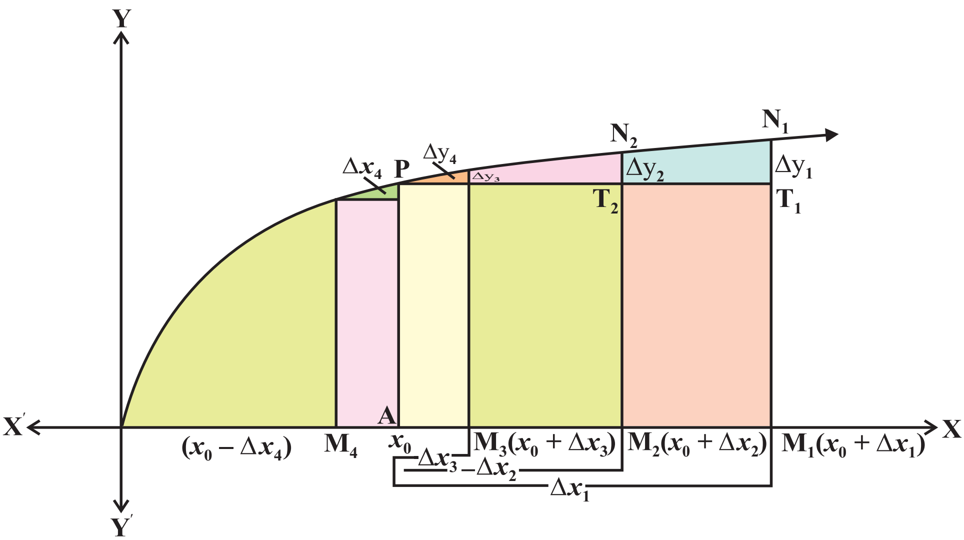

Fig. 10

DEMONSTRATION

1. Take one more point M_1 $(x_0 + \Delta x_1, 0)$ to the right of A, where Δx_1 is an increment in x.
2. Draw the perpendicular from M_1 to meet the curve at N_1. Let the coordinates of N_1 be $(x_0 + \Delta x_1, y_0 + \Delta y_1)$
3. Draw a perpendicular from the point P (x_0, y_0) to meet N_1M_1 at T_1.
4. Now measure $AM_1 = \Delta x_1$ (say) and record it and also measure $N_1T_1 = \Delta y_1$ and record it.
5. Reduce the increment in x to Δx_2 (i.e., $\Delta x_2 < \Delta x_1$) to get another point $M_2\left(x_0 + \Delta x_2, 0\right)$. Get the corresponding point N_2 on the curve
6. Let the perpendicular PT_1 intersects N_2M_2 at T_2.
7. Again measure $AM_2 = \Delta x_2$ and record it.

   Measure $N_2T_2 = \Delta y_2$ and record it.
8. Repeat the above steps for some more points so that Δx becomes smaller and smaller.

OBSERVATION

S.No.	Value of increment in x_0	Corresponding increment in y
1.	$\lvert\Delta x_1\rvert=$ ________	$\lvert\Delta y_1\rvert=$ ________
2.	$\lvert\Delta x_2\rvert=$ ________	$\lvert\Delta y_2\rvert=$ ________
3.	$\lvert\Delta x_3\rvert=$ ________	$\lvert\Delta y_3\rvert=$ ________
4.	$\lvert\Delta x_4\rvert=$ ________	$\lvert\Delta y_4\rvert=$ ________
5.	$\lvert\Delta x_5\rvert=$ ________	$\lvert\Delta y_5\rvert=$ ________

6.	$\|\Delta x_6\| =$	$\|\Delta y_6\| =$
7.	$\|\Delta x_7\| =$	$\|\Delta y_7\| =$
8.	$\|\Delta x_8\| =$	$\|\Delta y_8\| =$
9.	$\|\Delta x_9\| =$	$\|\Delta y_9\| =$

2. So, Δy becomes __________ when Δx becomes smaller.

3. Thus $\lim_{\Delta x \to 0} \Delta y = 0$ for a continuous function.

APPLICATION

This activity is helpful in explaining the concept of derivative (left hand or right hand) at any point on the curve corresponding to a function.

Activity 14

OBJECTIVE

To understand the concepts of local maxima, local minima and point of inflection.

MATERIAL REQUIRED

A piece of plywood, wires, adhesive, white paper.

METHOD OF CONSTRUCTION

1. Take a piece of plywood of a convenient size and paste a white paper on it.
2. Take two pieces of wires each of length 40 cm and fix them on the paper on plywood in the form of x-axis and y-axis.
3. Take another wire of suitable length and bend it in the shape of curve. Fix this curved wire on the white paper pasted on plywood, as shown in Fig. 14.

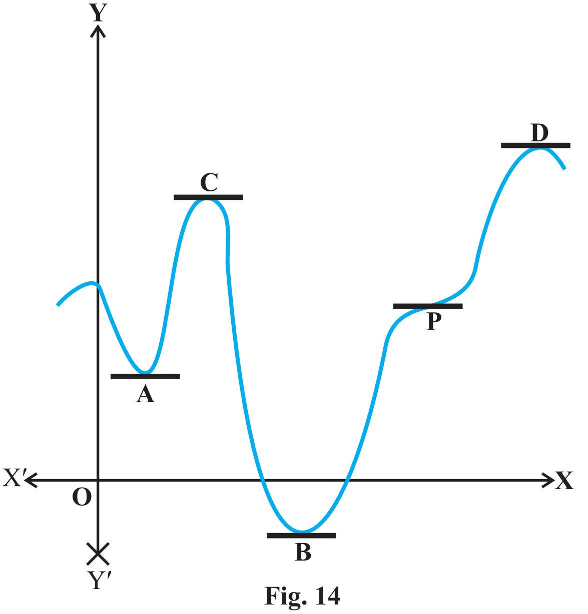

Fig. 14

4. Take five more wires each of length say 2 cm and fix them at the points A, C, B, P and D as shown in figure.

Demonstration

1. In the figure, wires at the points A, B, C and D represent tangents to the curve and are parallel to the axis. The slopes of tangents at these points are zero, i.e., the value of the first derivative at these points is zero. The tangent at P intersects the curve.
2. At the points A and B, sign of the first derivative changes from negative to positive. So, they are the points of local minima.
3. At the point C and D, sign of the first derivative changes from positive to negative. So, they are the points of local maxima.
4. At the point P, sign of first derivative does not change. So, it is a point of inflection.

Observation

1. Sign of the slope of the tangent (first derivative) at a point on the curve to the immediate left of A is _______.
2. Sign of the slope of the tangent (first derivative) at a point on the curve to the immediate right of A is_______.
3. Sign of the first derivative at a point on the curve to immediate left of B is _______.
4. Sign of the first derivative at a point on the curve to immediate right of B is _______.
5. Sign of the first derivative at a point on the curve to immediate left of C is _______.
6. Sign of the first derivative at a point on the curve to immediate right of C is _______.
7. Sign of the first derivative at a point on the curve to immediate left of D is _______.

8. Sign of the first derivative at a point on the curve to immediate right of D is _______.
9. Sign of the first derivative at a point immediate left of P is _______ and immediate right of P is_______.
10. A and B are points of local _______.
11. C and D are points of local _______.
12. P is a point of _______.

Application

1. This activity may help in explaining the concepts of points of local maxima, local minima and inflection.
2. The concepts of maxima/minima are useful in problems of daily life such as making of packages of maximum capacity at minimum cost.

Activity 16

OBJECTIVE

To construct an open box of maximum volume from a given rectangular sheet by cutting equal squares from each corner.

MATERIAL REQUIRED

Chart papers, scissors, cellotape, calculator.

METHOD OF CONSTRUCTION

1. Take a rectangular chart paper of size 20 cm × 10 cm and name it as ABCD.
2. Cut four equal squares each of side x cm from each corner A, B, C and D.
3. Repeat the process by taking the same size of chart papers and different values of x.
4. Make an open box by folding its flaps using cellotape/adhesive.

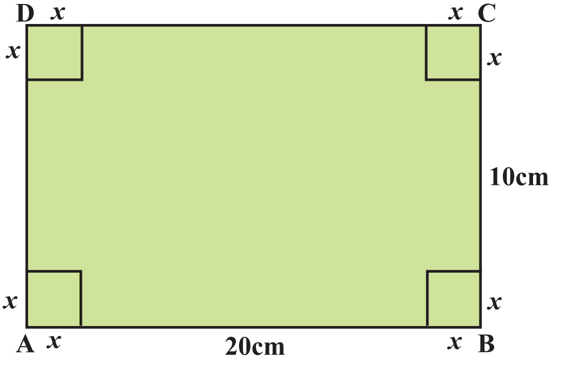

Fig. 16

DEMONSTRATION

1. When $x = 1$, Volume of the box = 144 cm^3
2. When $x = 1.5$, Volume of the box = 178.5 cm^3

3. When $x = 1.8$, Volume of the box = 188.9 cm^3.
4. When $x = 2$, Volume of the box = 192 cm^3.
5. When $x = 2.1$, Volume of the box = 192.4 cm^3.
6. When $x = 2.2$, Volume of the box = 192.2 cm^3.
7. When $x = 2.5$, Volume of the box = 187.5 cm^3.
8. When $x = 3$, Volume of the box = 168 cm^3.

Clearly, volume of the box is maximum when $x = 2.1$.

Observation

1. V_1 = Volume of the open box (when $x = 1.6$) = ...
2. V_2 = Volume of the open box (when $x = 1.9$) = ...
3. V = Volume of the open box (when $x = 2.1$) = ...
4. V_3 = Volume of the open box (when $x = 2.2$) = ...
5. V_4 = Volume of the open box (when $x = 2.4$) = ...
6. V_5 = Volume of the open box (when $x = 3.2$) = ...
7. Volume V_1 is _____________ than volume V.
8. Volume V_2 is _____________ than volume V.
9. Volume V_3 is _____________ than volume V.
10. Volume V_4 is _____________ than volume V.
11. Volume V_5 is _____________ than volume V.

So, Volume of the open box is maximum when x = ________.

Application

This activity is useful in explaining the concepts of maxima/minima of functions. It is also useful in making packages of maximum volume with minimum cost.

NOTE

Let V denote the volume of the box.

Now V = (20 – 2x) (10 – 2x) x

or V = 200x – 60x^2 + 4x^3

$\frac{dV}{dx} = 200 - 120x + 12x^2$. For maxima or minima, we have,

$\frac{dV}{dx} = 0$, i.e., $3x^2 - 30x + 50 = 0$

i.e., $x = \frac{30 \pm \sqrt{900 - 600}}{6} = 7.9$ or 2.1

Reject x = 7.9.

$\frac{d^2V}{dx^2} = -120 + 24x$

When x = 2.1, $\frac{d^2V}{dx^2}$ is negative.

Hence, V should be maximum at x = 2.1.

Activity 18

Objective

To verify that amongst all the rectangles of the same perimeter, the square has the maximum area.

Material Required

Chart paper, paper cutter, scale, pencil, eraser cardboard, glue.

Method of Construction

1. Take a cardboard of a convenient size and paste a white paper on it.
2. Make rectangles each of perimeter say 48 cm on a chart paper. Rectangles of different dimensions are as follows:

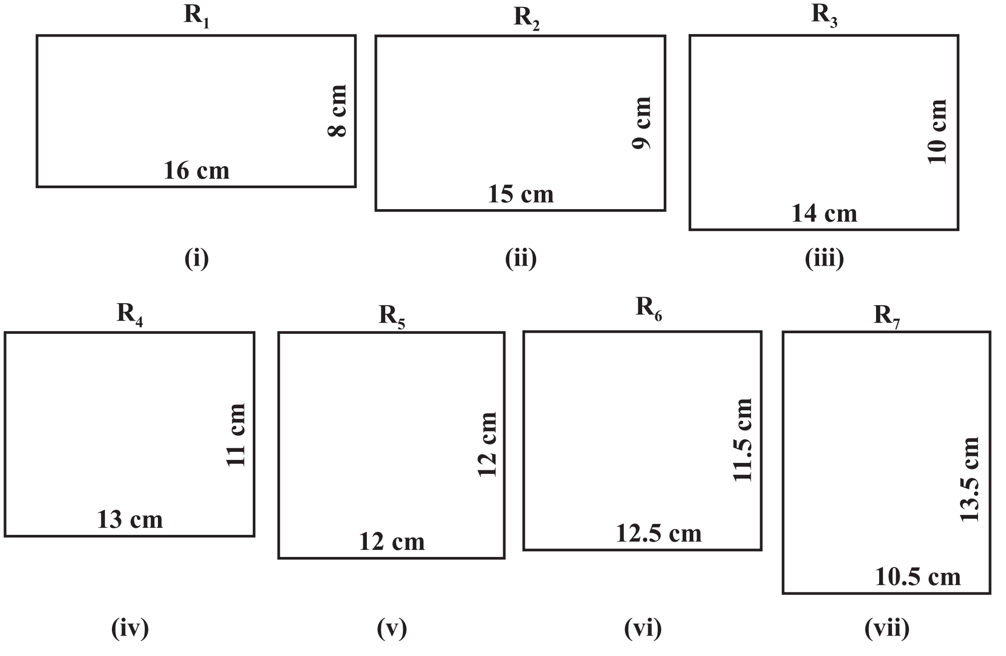

Fig. 18

R_1 : 16 cm × 8 cm, R_2 : 15 cm × 9 cm

R_3 : 14 cm × 10 cm, R_4 : 13 cm × 11 cm

R_5 : 12 cm × 12 cm, R_6 : 12.5 cm × 11.5 cm

R_7 : 10.5 cm × 13.5 cm

3. Cut out these rectangles and paste them on the white paper on the cardboard (see Fig. 18 (i) to (vii)).
4. Repeat step 2 for more rectangles of different dimensions each having perimeter 48 cm.
5. Paste these rectangles on cardboard.

DEMONSTRATION

1. Area of rectangle of R_1 = 16 cm × 8 cm = 128 cm^2

   Area of rectangle R_2 = 15 cm × 9 cm = 135 cm^2

   Area of R_3 = 140 cm^2

   Area of R_4 = 143 cm^2

   Area of R_5 = 144 cm^2

   Area of R_6 = 143.75 cm^2

   Area of R_7 = 141.75 cm^2

2. Perimeter of each rectangle is same but their area are different. Area of rectangle R_5 is the maximum. It is a square of side 12 cm. This can be verified using theoretical description given in the note.

OBSERVATION

1. Perimeter of each rectangle R_1, R_2, R_3, R_4, R_4, R_6, R_7 is _________.
2. Area of the rectangle R_3 ________ than the area of rectangle R_5.

3. Area of the rectangle R_6 _______ than the area of rectangle R_5.
4. The rectangle R_5 has the diamensions ______ × ______ and hence it is a ________.
5. Of all the rectangles with same perimeter, the ________ has the maximum area.

APPLICATION

This activity is useful in explaining the idea of Maximum of a function. The result is also useful in preparing economical packages.

NOTE

Let the length and breadth of rectangle be x and y.

The perimeter of the rectangle P = 48 cm.

$2(x + y) = 48$

or $x + y = 24$ or $y = 24 - x$

Let A (x) be the area of rectangle, then

$A(x) = xy$

$= x(24 - x)$

$= 24x - x^2$

$A'(x) = 24 - 2x$

$A'(x) = \Rightarrow 24 - 2x = 0 \Rightarrow x = 12$

$A''(x) = -2$

$A''(12) = -2$, which is negative

Therefore, area is maximum when $x = 12$

$y = x = 24 - 12 = 12$

So, $x = y = 12$

Hence, amongst all rectangles, the square has the maximum area.

Activity 20

OBJECTIVE

To verify geometrically that

$\vec{c}\times(\vec{a}+\vec{b})=\vec{c}\times\vec{a}+\vec{c}\times\vec{b}$

MATERIAL REQUIRED

Geometry box, cardboard, white paper, cutter, sketch pen, cellotape.

METHOD OF CONSTRUCTION

1. Fix a white paper on the cardboard.
2. Draw a line segment OA (= 6 cm, say) and let it represent $\vec{c}$.
3. Draw another line segment OB (= 4 cm, say) at an angle (say 60°) with OA.

   Let $\overrightarrow{OB}=\vec{a}$

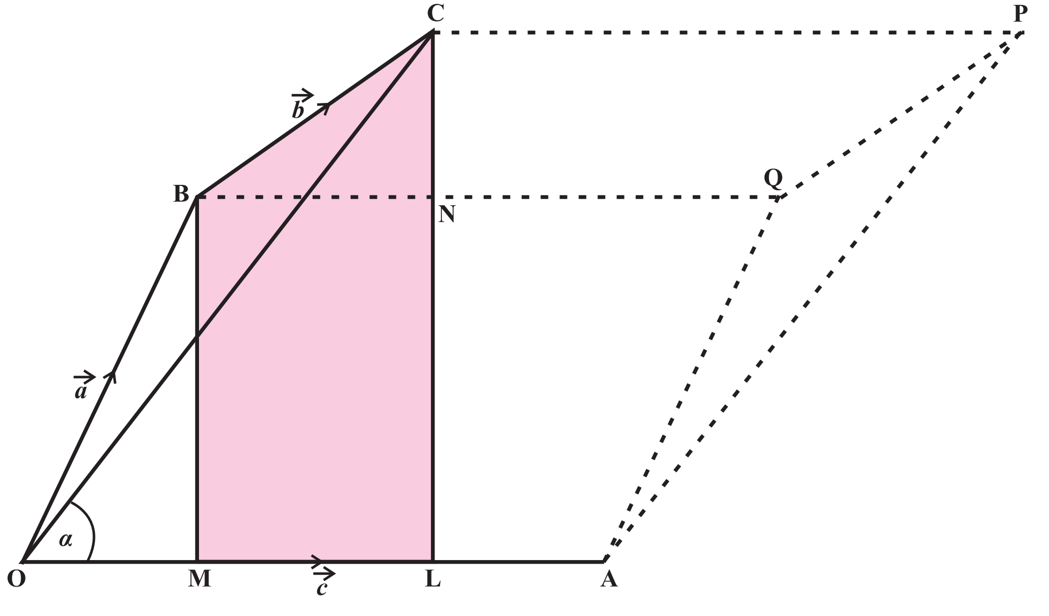

Fig. 20

4. Draw BC (= 3 cm, say) making an angle (say 30°) with $\overrightarrow{OA}$. Let $\overrightarrow{BC} = \vec{b}$
5. Draw perpendiculars BM, CL and BN.
6. Complete parallelograms OAPC, OAQB and BQPC.

DEMONSTRATION

1. $\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC} = \vec{a} + \vec{b}$, and let $\angle COA = \alpha$.

2. $\left|\vec{c} \times \left(\vec{a} + \vec{b}\right)\right| = \left|\vec{c}\right| \left|\vec{a} + \vec{b}\right|$ sin α = area of parallelogram OAPC.

3. $\left|\vec{c} \times \vec{a}\right|$ = area of parallelogram OAQB.

4. $\left|\vec{c} \times \vec{b}\right|$ = area of parallelogram BQPC.

5. Area of parallelogram OAPC = (OA) (CL)

= (OA) (LN + NC) = (OA) (BM + NC)

= (OA) (BM) + (OA) (NC)

= Area of parallelogram OAQB + Area of parallelogram BQPC

$= \left|\vec{c} + \vec{a}\right| + \left|\vec{c} \times \vec{b}\right|$

So, $\left|\vec{c} \times (\vec{a} + \vec{b})\right| = \left|\vec{c} \times \vec{b}\right| + \left|\vec{c} \times \vec{b}\right|$

Direction of each of these vectors $\vec{c} \times (\vec{a} + \vec{b})$, $\vec{c} \times \vec{a}$ and $\vec{c} \times \vec{b}$ is perpendicular to the same plane.

So, $\vec{c} \times (\vec{a} + \vec{b}) = \vec{c} \times \vec{a} + \vec{c} \times \vec{b}$.

OBSERVATION

$\left|\vec{c}\right| = \left|\overrightarrow{OA}\right| = OA =$ _______

$\left|\vec{a}+\vec{b}\right| = \left|\overrightarrow{OC}\right| = OC =$ ______

CL = ______

$\left|\vec{c}\times(\vec{a}+\vec{b})\right|$ = Area of parallelogram OAPC

= (OA) (CL) = _____________ sq. units (i)

$\left|\vec{c}\times\vec{a}\right|$ = Area of parallelogram OAQB

= (OA) (BM) = _____ × _____ = ______ (ii)

$\left|\vec{c}\times\vec{b}\right|$ = Area of parallelogram BQPC

= (OA) (CN) = _____ × _____ = ______ (iii)

From (i), (ii) and (iii),

Area of parallelogram OAPC = Area of parallelgram OAQB + Area of Parallelgram ________.

Thus $\left|\vec{c}\times\right|(\vec{a}+\vec{b}\,| = \left|\vec{c}\times\vec{a}\right| + \left|\vec{c}\times\vec{b}\right|$

$\vec{c}\times\vec{a}$, $\vec{c}\times\vec{b}$ and $\vec{c}\times\left(\vec{a}+\vec{b}\right)$ are all in the direction of _______ to the plane of paper.

Therefore $\vec{c}\times\left(\vec{a}+\vec{b}\right) = \vec{c}\times\vec{a} +$ ________.

APPLICATION

Through the activity, distributive property of vector multiplication over addition can be explained.

NOTE

This activity can also be performed by taking rectangles instead of parallelograms.

Activity 21

Objective

To verify that angle in a semi-circle is a right angle, using vector method.

Material Required

Cardboard, white paper, adhesive, pens, geometry box, eraser, wires, paper arrow heads.

Method of Construction

1. Take a thick cardboard of size 30 cm × 30 cm.
2. On the cardboard, paste a white paper of the same size using an adhesive.
3. On this paper draw a circle, with centre O and radius 10 cm.

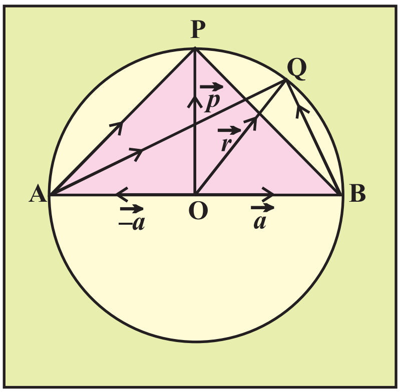

Fig. 21

4. Fix nails at the points O, A, B, P and Q. Join OP, OA, OB, AP, AQ, BQ, OQ and BP using wires.
5. Put arrows on OA, OB, OP, AP, BP, OQ, AQ and BQ to show them as vectors, using paper arrow heads, as shown in the figure.

Demonstration

1. Using a protractor, measure the angle between the vectors $\overrightarrow{AP}$ and $\overrightarrow{BP}$, i.e., $\angle APB = 90°$.

2. Similarly, the angle between the vectors $\overrightarrow{AQ}$ and $\overrightarrow{BQ}$, i.e., $\angle$ AQB = 90°.

3. Repeat the above process by taking some more points R, S, T, ... on the semi-circles, forming vectors AR, BR; AS, BS; AT, BT; ..., etc., i.e., angle formed between two vectors in a semi-circle is a right angle.

Observation

By actual measurement.

$|\overrightarrow{OP}| = |\overrightarrow{OA}| = |\overrightarrow{OB}| = |\overrightarrow{OQ}| = r = a = p =$ _______ ,

$|\overrightarrow{AP}|$ = _______ , $|\overrightarrow{BP}|$ = _______, $|\overrightarrow{AB}|$ = ______

$|\overrightarrow{AQ}|$ = _______ , $|\overrightarrow{BQ}|$ = _______

$|\overrightarrow{AP}|^2 + |\overrightarrow{BP}|^2$ = ________, $|\overrightarrow{AQ}|^2 + |\overrightarrow{BQ}|^2$ = ________

So, $\angle$APB = ________ and $\overrightarrow{AP}.\overrightarrow{BP}$ ________ $\angle$AQB = ________ and

$\overrightarrow{AQ}.\overrightarrow{BP}$ = ________

Similarly, for points R, S, T, ________

$\angle$ARB = ________, $\angle$ASB = ________, $\angle$ATB = ________, ________

i.e., angle in a semi-circle is a right angle.

Application

This activity can be used to explain the concepts of

(i) opposite vectors

(ii) vectors of equal magnitude

(iii) perpendicular vectors

(iv) Dot product of two vectors.

NOTE

Let OA = OB = a = OP = p

$\overrightarrow{OA} = -\vec{a}$, $\overrightarrow{OB} = \vec{a}$, $\overrightarrow{OP} = \vec{p}$

$\overrightarrow{AP} = -\overrightarrow{OA} + \overrightarrow{OP} = \vec{a} + \vec{p}$., $\overrightarrow{BP} = \vec{p} - \vec{a}$.

$$\overrightarrow{AP}.\overrightarrow{BP} = (\vec{p}+\vec{a}).(\vec{p}-\vec{a}) = |\vec{p}|^2 - |\vec{a}|^2 = 0$$

$$\left(\text{since} |\vec{p}|^2 = |\vec{a}|\right)$$

So, the angle APB between the vectors $\overrightarrow{AP}$ and $\overrightarrow{BP}$ is a right angle.

Similarly, $\overrightarrow{AQ}.\overrightarrow{BQ} = 0$, so, $\angle AQB = 90°$ and so on.

Activity 26

Objective

To measure the shortest distance between two skew lines and verify it analytically.

Material Required

A piece of plywood of size 30 cm × 20 cm, a squared paper, three wooden blocks of size 2cm × 2 cm × 2 cm each and one wooden block of size 2 cm × 2 cm × 4 cm, wires of different lengths, set squares, adhesive, pen/pencil, etc.

Method of Construction

1. Paste a squared paper on a piece of plywood.
2. On the squared paper, draw two lines OA and OB to represent *x*-axis, and *y*-axis, respectively.
3. Name the three blocks of size 2 cm × 2 cm × 2 cm as I, II and III. Name the other wooden block of size 2 cm × 2 cm × 4 cm as IV.
4. Place blocks I, II, III such that their base centres are at the points (2, 2), (1, 6) and (7, 6), respectively, and block IV with its base centre at (6, 2). Other wooden block of size 2 cm × 2 cm × 4 cm as IV.
5. Place a wire joining the points P and Q, the centres of the bases of the blocks I and III and another wire joining the centres R and S of the tops of blocks II and IV as shown in Fig. 26.
6. These two wires represent two skew lines.
7. Take a wire and join it perpendicularly with the skew lines and measure the actual distance.

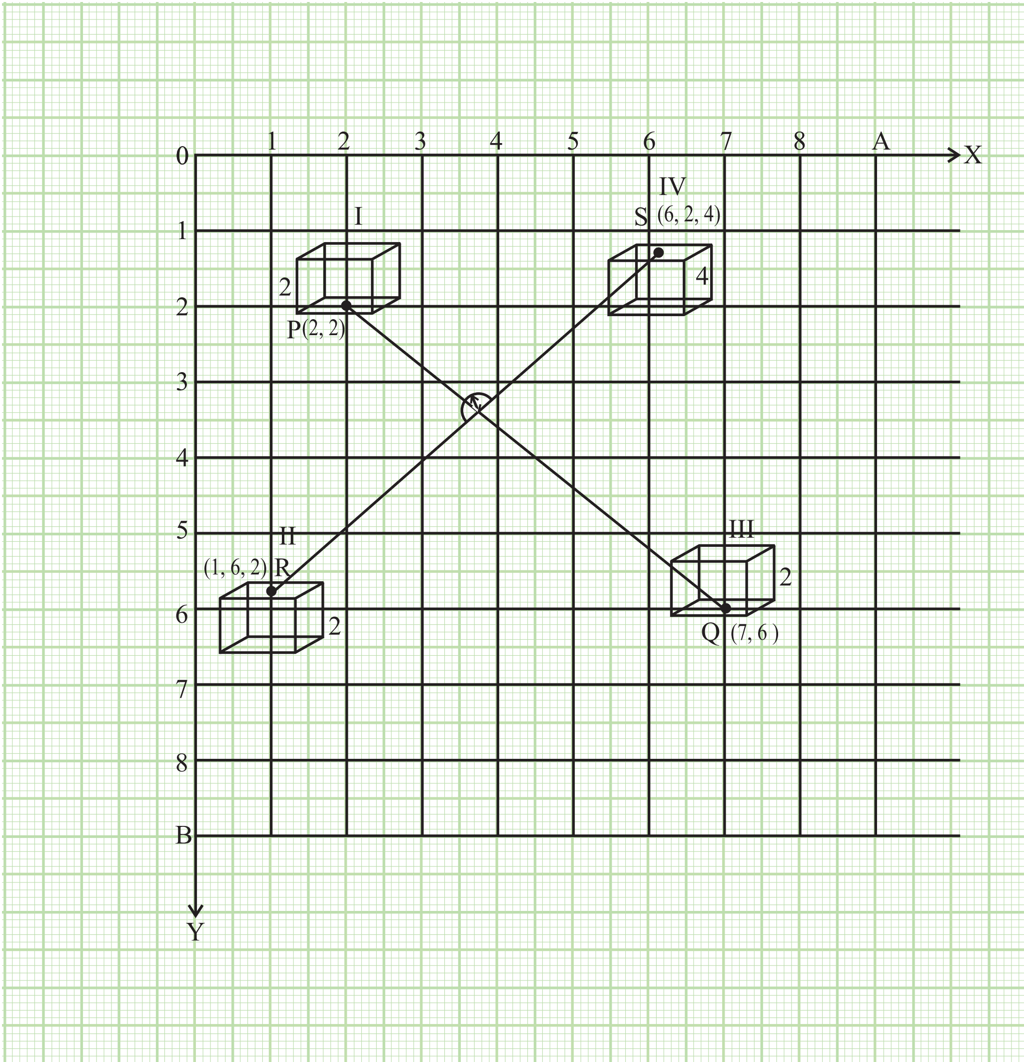

Fig. 26

Demonstration

1. A set-square is placed in such a way that its one perpendicular side is along the wire PQ.
2. Move the set-square along PQ till its other perpendicular side touches the other wire.

3. Measure the distance between the two lines in this position using set-square. This is the shortest distance between two skew lines.

4. Analytically, find the equation of line joining P (2, 2, 0) and Q (7, 6, 0) and other line joining R (1, 6, 2) and S (6, 2, 4) and find S.D. using $\frac{\left(\vec{a_2}-\vec{a_1}\right)\cdot\left(\vec{b_1}\times\vec{b_2}\right)}{\left|b_1\times\vec{b_2}\right|}$. The distance obtained in two cases will be the same.

OBSERVATION

1. Coordinates of point P are ________.
2. Coordinates of point Q are ________.
3. Coordinates of point R are ________.
4. Coordinates of point S are ________.
5. Equation of line PQ is ________.
6. Equation of line RS is ________.

Shortest distance between PQ and RS analytically = ________.

Shortest distance by actual measurement = ________.

The results so obtained are ________.

APPLICATION

This activity can be used to explain the concept of skew lines and of shortest distance between two lines in space.

Activity 27

Objective

To explain the computation of conditional probability of a given event A, when event B has already occurred, through an example of throwing a pair of dice.

Material Required

A piece of plywood, white paper pen/pencil, scale, a pair of dice.

Method of Construction

1. Paste a white paper on a piece of plywood of a convenient size.
2. Make a square and divide it into 36 unit squares of size 1cm each (see Fig. 27).
3. Write pair of numbers as shown in the figure.

1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

Fig. 27

Demonstration

1. Fig. 27 gives all possible outcomes of the given experiment. Hence, it represents the sample space of the experiment.
2. Suppose we have to find the conditional probability of an event A if an event B has already occurred, where A is the event "a number 4 appears on both the dice" and B is the event "4 has appeared on at least one of the dice" i.e, we have to find P(A | B).
3. From Fig. 27 number of outcomes favourable to A = 1

   Number of outcomes favourable to B = 11

   Number of outcomes favourable to $A \cap B$ = 1.

4. (i) $P(B) = \frac{11}{36}$,

   (ii) $P(A \cap B) = \frac{1}{36}$

   (iii) $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{1}{11}$.

Note

1. You may repeat this activity by taking more events such as the probability of getting a sum 10 when a doublet has already occurred.

2. Conditional probability P (A | B) can also be found by first taking the sample space of event B out of the sample space of the experiment, and then finding the probability A from it.

Observation

1. Outcome(s) favourable to A : __________, n (A) = __________.
2. Outcomes favourable to B : __________, n (B) = __________.
3. Outcomes favourable to $A \cap B$: __________, $n(A \cap B)$ = __________.
4. $P(A \cap B)$ = __________.
5. P (A | B) = __________ = __________.

Application

This activity is helpful in understanding the concept of conditional probability, which is further used in Bayes' theorem.