Activity 3

OBJECTIVE

To represent set theoretic operations using Venn diagrams.

MATERIAL REQUIRED

Hardboard, white thick sheets of paper, pencils, colours, scissors, adhesive.

METHOD OF CONSTRUCTION

1. Cut rectangular strips from a sheet of paper and paste them on a hardboard. Write the symbol U in the left/right top corner of each rectangle.
2. Draw circles A and B inside each of the rectangular strips and shade/colour different portions as shown in Fig. 3.1 to Fig. 3.10.

DEMONSTRATION

1. U denotes the universal set represented by the rectangle.
2. Circles A and B represent the subsets of the universal set U as shown in the figures 3.1 to 3.10.
3. A' denote the complement of the set A, and B' denote the complement of the set B as shown in the Fig. 3.3 and Fig. 3.4.
4. Coloured portion in Fig. 3.1. represents $A \cup B$.

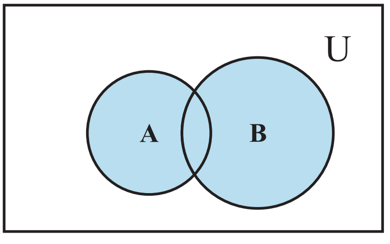

Fig. 3.1

5. Coloured portion in Fig. 3.2. represents A ∩ B.

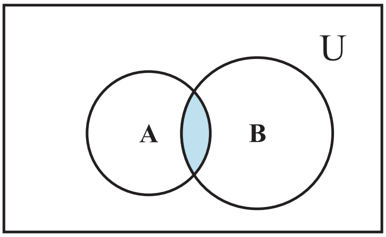

Fig. 3.2

6. Coloured portion in Fig. 3.3 represents A′

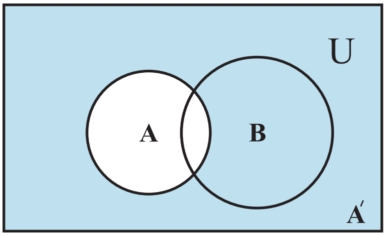

Fig. 3.3

7. Coloured portion in Fig. 3.4 represents B′

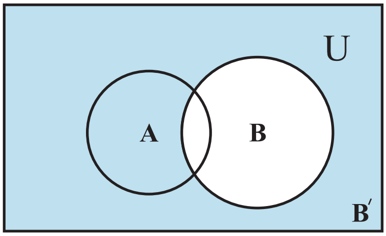

Fig. 3.4

8. Coloured portion in Fig. 3.5 represents (A ∩ B)′

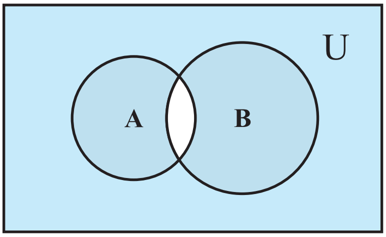

Fig. 3.5

9. Coloured portion in Fig. 3.6 represents $(A \cup B)'$

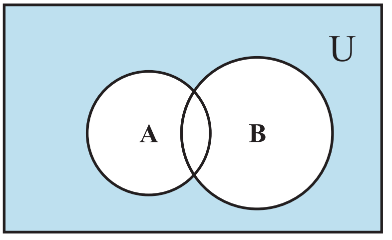

Fig. 3.6

10. Coloured portion in Fig. 3.7 represents $A' \cap B$ which is same as B – A.

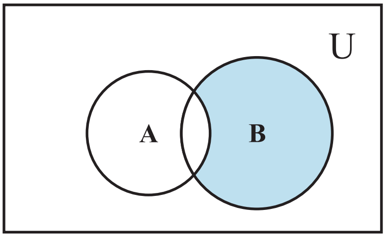

Fig. 3.7

11. Coloured portion in Fig. 3.8 represents $A' \cup B$.

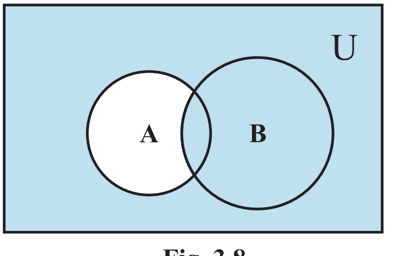

Fig. 3.8

12. Fig. 3.9 shows $A \cap B = \phi$

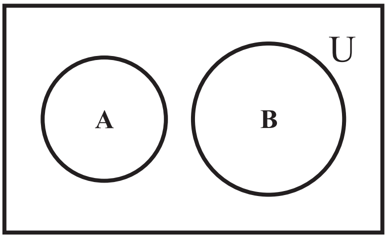

Fig. 3.9

13. Fig. 3.10 shows $A \subset B$

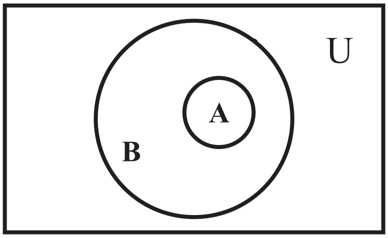

Fig. 3.10

Observation

1. Coloured portion in Fig. 3.1, represents ______________
2. Coloured portion in Fig. 3.2, represents ______________
3. Coloured portion in Fig. 3.3, represents ______________
4. Coloured portion in Fig. 3.4, represents ______________
5. Coloured portion in Fig. 3.5, represents ______________
6. Coloured portion in Fig. 3.6, represents ______________
7. Coloured portion in Fig. 3.7, represents ______________
8. Coloured portion in Fig. 3.8, represents ______________
9. Fig. 3.9, shows that $(A \cap B)$ = ______________
10. Fig. 3.10, represents A ______________ B.

Application

Set theoretic representation of Venn diagrams are used in Logic and Mathematics.

Activity 6

OBJECTIVE

To distinguish between a Relation and a Function.

MATERIAL REQUIRED

Drawing board, coloured drawing sheets, scissors, adhesive, strings, nails etc.

METHOD OF CONSTRUCTION

1. Take a drawing board/a piece of plywood of convenient size and paste a coloured sheet on it.
2. Take a white drawing sheet and cut out a rectangular strip of size 6 cm × 4 cm and paste it on the left side of the drawing board (see Fig. 6.1).

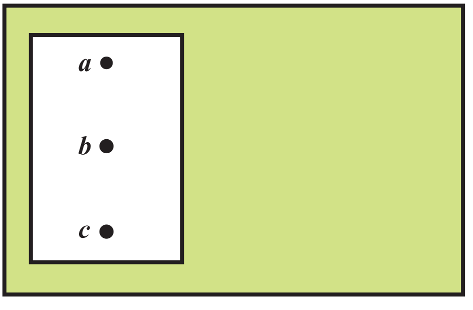

Fig. 6.1

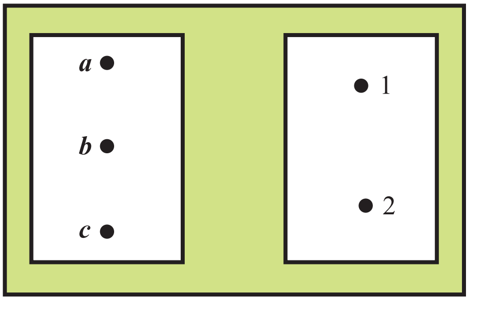

Fig. 6.2

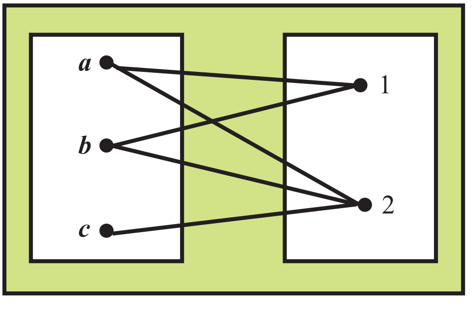

Fig. 6.3

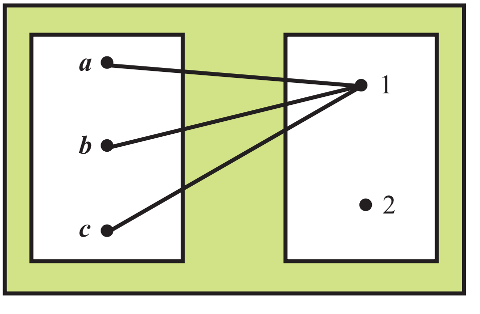

Fig. 6.4

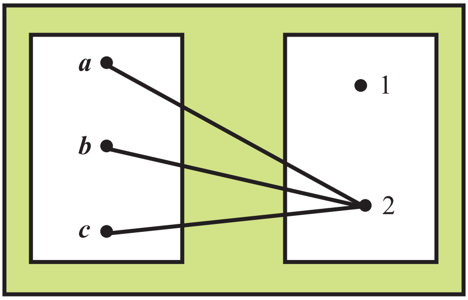

Fig. 6.5

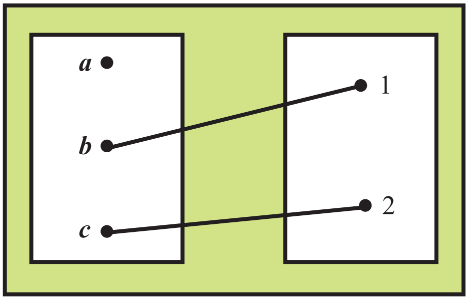

Fig. 6.6

3. Fix three nails on this strip and mark them as *a, b, c* (see Fig. 6.1).
4. Cut out another white rectangular strip of size 6 cm × 4 cm and paste it on the right hand side of the drawing board.
5. Fix two nails on the right side of this strip (see Fig. 6.2) and mark them as 1 and 2.

Demonstration

1. Join nails of the left hand strip to the nails on the right hand strip by strings in different ways. Some of such ways are shown in Fig. 6.3 to Fig. 6.6.
2. Joining nails in each figure constitute different ordered pairs representing elements of a relation.

Observation

1. In Fig. 6.3, ordered pairs are ____________.

   These ordered pairs constitute a ___________ but not a _________.
2. In Fig. 6.4, ordered pairs are __________. These constitute a _______ as well as ________.
3. In Fig 6.5, ordered pairs are _______. These ordered pairs constitute a ________ as well as ________.
4. In Fig. 6.6, ordered pairs are ________. These ordered pairs do not represent ______ but represent ________.

APPLICATION

Such activity can also be used to demonstrate different types of functions such as constant function, identity function, injective and surjective functions by joining nails on the left hand strip to that of right hand strip in suitable manner.

NOTE

In the above activity nails have been joined in some different ways. The student may try to join them in other different ways to get more relations of different types. The number of nails can also be changed on both sides to represent different types of relations and functions.

Activity 8

Objective

To find the values of sine and cosine functions in second, third and fourth quadrants using their given values in first quadrant.

Material Required

Cardboard, white chart paper, ruler, coloured pens, adhesive, steel wires and needle.

Method of Construction

1. Take a cardboard of convenient size and paste a white chart paper on it.
2. Draw a unit circle with centre O on chart paper.
3. Through the centre of the circle, draw two perpendicular lines $X'OX$ and YOY' representing *x*-axis and *y*-axis, respectively, as shown in Fig.8.1.

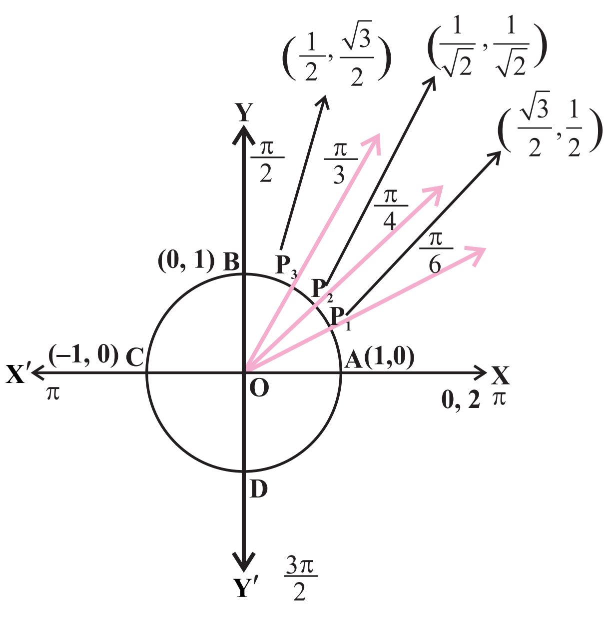

Fig. 8.1

4. Mark the points as A, B, C and D, where the circle cuts the x-axis and y-axis, respectively, as shown in Fig. 8.1.

5. Through O, draw angles P_1OX, P_2OX, and P_3OX of measures $\frac{\pi}{6}, \frac{\pi}{4}$ and $\frac{\pi}{3}$, respectively.

6. Take a needle of unit length. Fix one end of it at the centre of the circle and the other end to move freely along the circle.

Demonstration

1. The coordinates of the point P_1 are $\left(\sqrt{\frac{3}{2}}, \frac{1}{2}\right)$ because its x-coordinate is $\cos \frac{\pi}{6}$ and y-coordinate is $\sin \frac{\pi}{6}$. The coordinates of the points P_2 and P_3 are $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, respectively.

2. To find the value of sine or cosine of some angle in the second quadrant (say) $\frac{2\pi}{3}$, rotate the needle in anti clockwise direction making an angle P_4OX of measure $\frac{2\pi}{3} = 120°$ with the positive direction of x-axis.

3. Look at the position OP_4 of the needle in

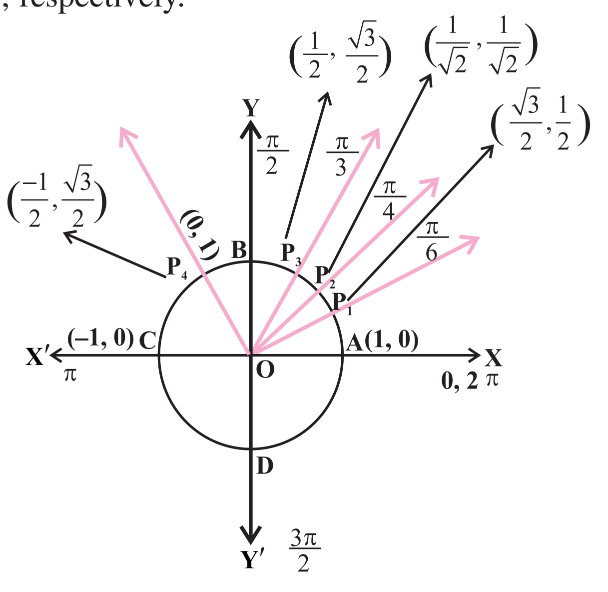

Fig. 8.2

Fig.8.2. Since $\frac{2\pi}{3} = \pi - \frac{\pi}{3}$, OP_4 is the mirror image of OP_3 with respect to y-axis. Therefore, the coordinate of P_4 are $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$. Thus $\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$ and $\cos\frac{2\pi}{3} = -\frac{1}{2}$.

4. To find the value of sine or cosine of some angle say, $\pi + \frac{\pi}{3} = \frac{4\pi}{3}$, i.e., $\frac{-2\pi}{3}$ (say) in the third quadrant, rotate the needle in anti clockwise direction making as an angle of $\frac{4\pi}{3}$ with the positive direction of x-axis.

5. Look at the new position OP_5 of the needle, which is shown in Fig. 8.3. Point P_5 is the mirror image of the point P_4 (since $\angle P_4OX' = P_5OX'$) with respect to x-axis. Therefore, co-ordinates of P_5 are $\left(-\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$ and hence

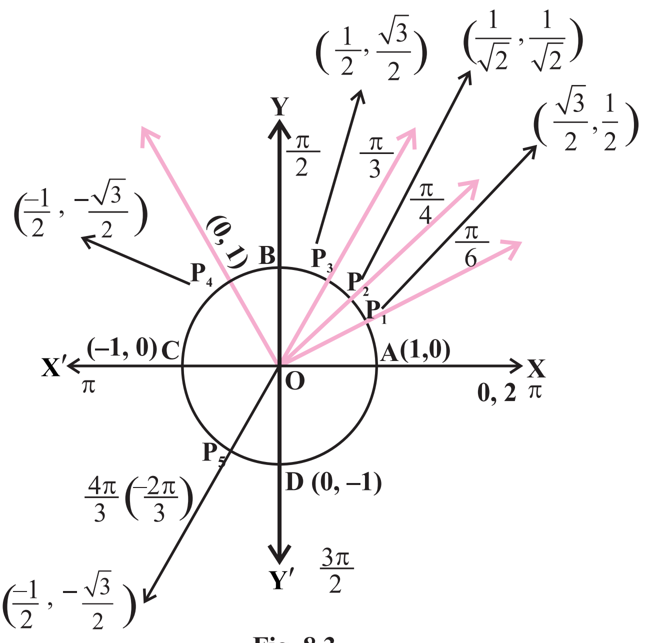

Fig. 8.3

$$\sin\left(-\frac{2\pi}{3}\right) = \sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2} \text{ and } \cos\left(-\frac{2\pi}{3}\right) = \cos\left(\frac{4\pi}{3}\right) = -\frac{1}{2}.$$

6. To find the value of sine or cosine of some angle in the fourth quadrant, say $\frac{7\pi}{4}$, rotate the needle in anti clockwise direction making an angle of $\frac{7\pi}{4}$ with the positive direction of x-axis represented by OP_6, as shown in Fig. 8.4. Angle $\frac{7\pi}{4}$ in anti clockwise direction = Angle $-\frac{\pi}{4}$ in the clockwise direction.

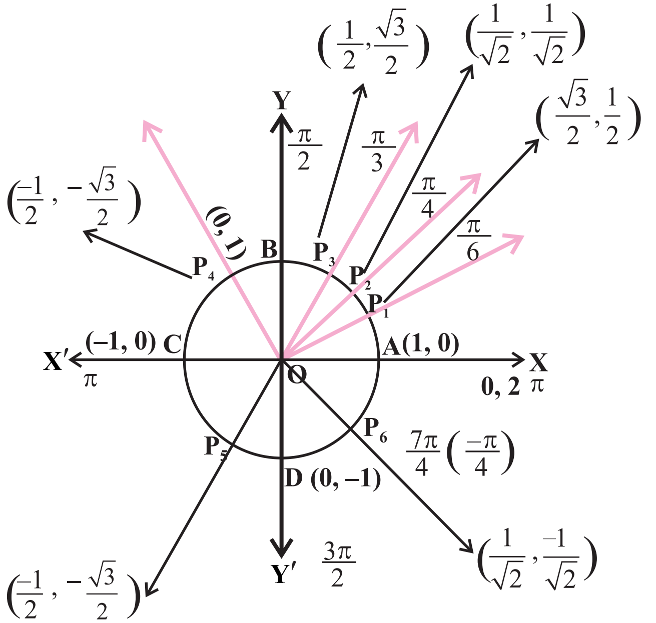

Fig. 8.4

From Fig. 8.4, P_6 is the mirror image of P_2 with respect to x-axis. Therefore, coordinates of P_6 are $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$.

Thus $\sin\left(\frac{7\pi}{4}\right)=\sin\left(-\frac{\pi}{4}\right)=-\frac{1}{\sqrt{2}}$

and $\cos\left(\frac{7\pi}{4}\right)=\cos\left(-\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}$

8. To find the value of sine or cosine of some angle, which is greater than one revolution, say $\frac{13\pi}{6}$, rotate the needle in anti clockwise direction since $\frac{13\pi}{6}=2\pi+\frac{\pi}{6}$, the needle will reach at the position OP_1. Therefore,

   $\sin\left(\frac{13\pi}{6}\right)=\sin\left(\frac{\pi}{6}\right)=\frac{1}{2}$ and $\cos\left(\frac{13\pi}{6}\right)=\cos\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$.

OBSERVATION

1. Angle made by the needle in one complete revolution is _________.
2. $\cos\frac{\pi}{6}=$ _________ $=\cos\left(-\frac{\pi}{6}\right)$

   $\sin\frac{\pi}{6}=$ _________ $=\sin(2\pi+$ ______$)$.
3. sine function is non-negative in _______ and _______ quadrants.
4. cosine function is non-negative in ______ and _____ quadrants.

APPLICATION

1. The activity can be used to get the values for tan, cot, sec, and cosec functions also.
2. From this activity students may learn that
   $\sin(-\theta) = -\sin\theta$ and $\cos(-\theta) = -\cos\theta$

This activity can be applied to other trigonometric functions also.

Activity 9

OBJECTIVE

To prepare a model to illustrate the values of sine function and cosine function for different angles which are multiples of $\frac{\pi}{2}$ and π.

MATERIAL REQUIRED

A stand fitted with 0°-360° protractor and a circular plastic sheet fixed with handle which can be rotated at the centre of the protractor.

METHOD OF CONSTRUCTION

1. Take a stand fitted with 0º-360º protractor.
2. Consider the radius of protractor as 1 unit.

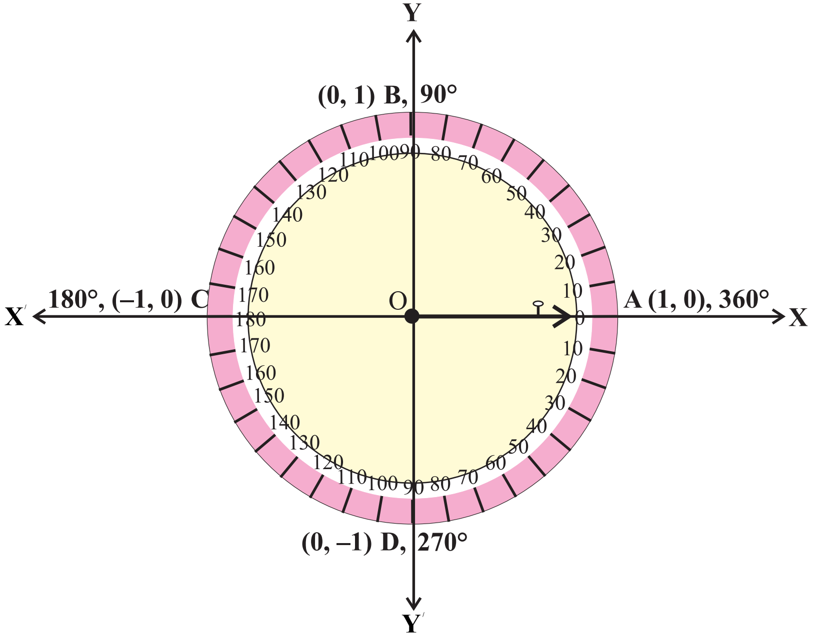

Fig. 9

3. Draw two lines, one joining 0º-180º line and another 90º-270º line, obviously perpendicular to each other.
4. Mark the ends of 0°-180° line as (1,0) at 0°, (–1, 0) at 180° and that of 90° - 270° line as (0,1) at 90° and (0, –1) at 270°
5. Take a plastic circular plate and mark a line to indicate its radius and fix a handle at the outer end of the radius.
6. Fix the plastic circular plate at the centre of the protractor.

DEMONSTRATION

1. Move the circular plate in anticlock wise direction to make different angles like 0, $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$, 2π etc.
2. Read the values of sine and cosine function for these angles and their multiples from the perpendicular lines.

OBSERVATION

1. When radius line of circular plate is at 0º indicating the point A (1,0), cos 0 = ______ and sin 0 = _______.
2. When radius line of circular plate is at 90º indicating the point B (0, 1), $\cos\frac{\pi}{2}$ = _______ and $\sin\frac{\pi}{2}$ = _______ .
3. When radius line of circular plate is at 180º indicating the point C (–1,0), cos π = ______ and sin π = _________.
4. When radius line of circular plate is at 270º indicating the point D (0, – 1) which means $\cos\frac{3\pi}{2}$ = _______ and $\sin\frac{3\pi}{2}$ = _______
5. When radius line of circular plate is at 360º indicating the point again at A (1,0), cos 2 π = _______ and sin 2 π = ________.

Now fill in the table :

Trigonometric function	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{5\pi}{2}$	3π	$\frac{7\pi}{2}$	4π
sin θ	–	–	–	–	–	–	–	–	–
cos θ	–	–	–	–	–	–	–	–	–

APPLICATION

This activity can be used to determine the values of other trigonometric functions for angles being multiple of $\frac{\pi}{2}$ and π.

Activity 10

OBJECTIVE

To plot the graphs of sin x, sin $2x$, $2\sin x$ and $\sin\frac{x}{2}$, using same coordinate axes.

MATERIAL REQUIRED

Plyboard, squared paper, adhesive, ruler, coloured pens, eraser.

METHOD OF CONSTRUCTION

1. Take a plywood of size 30 cm × 30 cm.
2. On the plywood, paste a thick graph paper of size 25 cm × 25 cm.
3. Draw two mutually perpendicular lines on the squared paper, and take them as coordinate axes.
4. Graduate the two axes as shown in the Fig. 10.
5. Prepare the table of ordered pairs for sin x, sin $2x$, 2sin x and sin $\frac{x}{2}$ for different values of x shown in the table below:

T. ratios	**0°**	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$	$\frac{7\pi}{12}$	$\frac{2\pi}{3}$	$\frac{9\pi}{12}$	$\frac{5\pi}{6}$	$\frac{11\pi}{12}$	π
sin x	0	0.26	0.50	0.71	0.86	0.97	1.00	0.97	0.86	0.71	0.50	0.26	0
sin $2x$	0	0.50	0.86	1.00	0.86	0.50	0	–0.5	–0.86	–1.0	–0.86	–0.50	0
2 sin x	0	0.52	1.00	1.42	1.72	1.94	2.00	1.94	1.72	1.42	1.00	0.52	0
sin $\frac{x}{2}$	0	0.13	0.26	0.38	0.50	0.61	0.71	0.79	0.86	0.92	0.97	0.99	1.00

DEMONSTRATION

1. Plot the ordered pair (x, sin x), (x, sin 2x), (x, sin $\frac{x}{2}$) and (x, 2sin x) on the same axes of coordinates, and join the plotted ordered pairs by free hand curves in different colours as shown in the Fig.10.

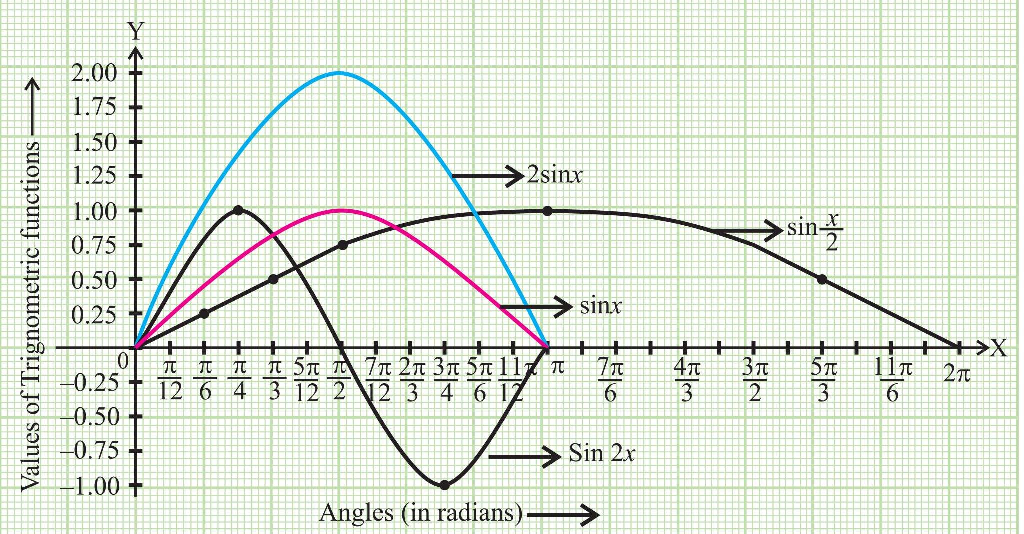

Fig. 10

OBSERVATION

1. Graphs of sin x and 2 sin x are of same shape but the maximum height of the graph of sin x is _________ the maximum height of the graph of ___________.
2. The maximum height of the graph of sin 2x is ____________. It is at x = __________.
3. The maximum height of the graph of 2 sin x is ____________ . It is at x = __________ .

4. The maximum height of the graph of sin $\frac{x}{2}$ is ____________ . It is at $\frac{x}{2}$ = __________ .

5. At x = _________, sin x = 0, at x = ________, sin $2x$ = 0 and at x = _________, sin $\frac{x}{2}$ = 0.

6. In the interval [0, π], graphs of sin x, 2 sin x and sin $\frac{x}{2}$ are ________ x - axes and some portion of the graph of sin $2x$ lies ________ x-axes.

7. Graphs of sin x and sin $2x$ intersect at x = ________ in the interval (0, π)

8. Graphs of sin x and sin $\frac{x}{2}$ intersect at x = _________ in the interval (0, π).

APPLICATION

This activity may be used in comparing graphs of a trigonometric function of multiples and submultiples of angles.

Activity 11

Objective

To inerpret geometrically the meaning of $i=\sqrt{-1}$ and its integral powers.

Material Required

Cardboard, chart paper, sketch pen, ruler, compasses, adhesive, nails, thread.

Method of Construction

1. Paste a chart paper on the cardboard of a convenient size.
2. Draw two mutually perpendicular lines $X'X$ and $Y'Y$ interesting at the point O (see Fig. 11).
3. Take a thread of a unit length representing the number 1 along OX . Fix one end of the thread to the nail at 0 and the other end at A as shown in the figure.
4. Set free the other end of the thread at A and rotate the thread through angles of 90º, 180º, 270º and 360º and mark the free end of the thread in different cases as A_1, A_2, A_3 and A_4 , respectively, as shown in the figure.

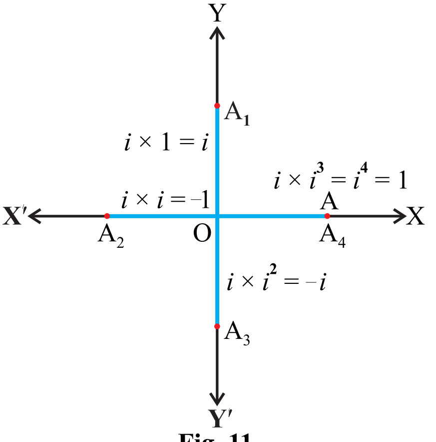

Fig. 11

Demonstration

1. In the argand plane, OA, OA_1, OA_2, OA_3, OA_4 represent, respectively, 1, i, –1, – i, 1.
2. $OA_1 = i = 1 \times i$, $OA_2 = -1 = i \times i = i^2$, $OA_3 = -i = i \times i \times i = i^3$ and so on. Each time, rotation of OA by 90º is equivalent to multiplication by i. Thus, i is referred to as the multiplying factor for a rotation of 90º.

Observation

1. On rotating OA through 90º, $OA_1 = 1 \times i =$ _________.
2. On rotating OA through an angle of 180°, $OA_2 = 1 \times$ __ $\times$ __ $=$ ______.
3. On rotation of OA through 270º (3 right angles), $OA_3 =$

   $1 \times$ _______ $\times$ ______$\times$ ______ $=$ ______.
4. On rotating OA through 360º (4 right angles),

   $OA_4 = 1 \times$ _______ $\times$ ______$\times$ ______ $\times$ ______ $=$ __________.
5. On rotating OA through n-right angles

   $OA_n = 1 \times$ _______ $\times$ ______$\times$ ______ $\times$ ______ $\times$ ... n times $=$ __________.

Application

This activity may be used to evaluate any integral power of i.

Activity 15

Objective

To construct a Pascal's Triangle and to write binomial expansion for a given positive integral exponent.

Material Required

Drawing board, white paper, matchsticks, adhesive.

Method of Construction

1. Take a drawing board and paste a white paper on it.
2. Take some matchsticks and arrange them as shown in Fig.15.

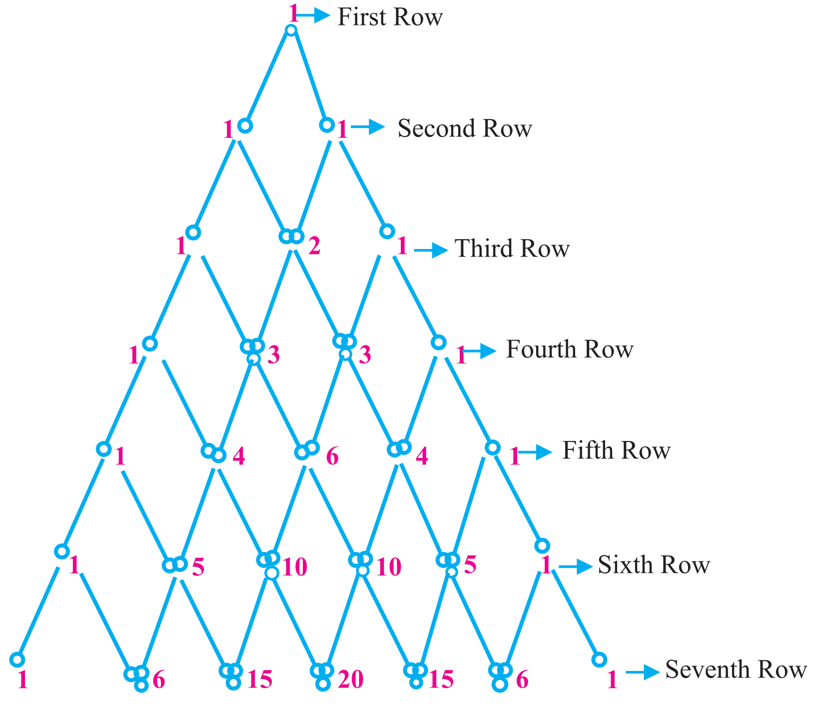

Fig. 15

3. Write the numbers as follows:

   1 (first row)
   1 1 (second row)
   1 2 1 (third row)
   1 3 3 1 (fourth row), 1 4 6 4 1 (fifth row) and so on (see Fig. 15).

4. To write binomial expansion of $(a + b)^n$, use the numbers given in the $(n + 1)^{th}$ row.

Demonstration

1. The above figure looks like a triangle and is referred to as Pascal's Triangle.
2. Numbers in the second row give the coefficients of the terms of the binomial expansion of $(a + b)^1$. Numbers in the third row give the coefficients of the terms of the binomial expansion of $(a + b)^2$, numbers in the fourth row give coefficients of the terms of binomial expansion of $(a + b)^3$. Numbers in the fifth row give coefficients of the terms of binomial expansion of $(a + b)^4$ and so on.

Observation

1. Numbers in the fifth row are ___________, which are coefficients of the binomial expansion of __________.
2. Numbers in the seventh row are _____________, which are coefficients of the binomial expansion of _______.
3. $(a + b)^3$ = ___ a^3 + ___a^2b + ___ab^2 + ___b^3
4. $(a + b)^5$ = ___ +___+ ___+ ___ + ___+ ___.
5. $(a + b)^6$ =___a^6 +___a^5b + ___a^4b^2 + ___a^3b^3 + ___a^2b^4 + ___ab^5 + ___b^6.
6. $(a + b)^8$ = ___ +___ +___+ ___ + ___+ ___ + ___ + ___+ ___.
7. $(a + b)^{10}$ =___ + ___ + ___+ ___ + ___+ ___ + ___+___+ ___+ ___+ __.

Application

The activity can be used to write binomial expansion for $(a + b)^n$, where n is a positive integer.

Activity 21

OBJECTIVE

To construct different types of conic sections.

MATERIAL REQUIRED

Transparent sheet, scissors, hard-board, adhesive, white paper.

METHOD OF CONSTRUCTION

1. Take a hardboard of convenient size and paste a white paper on it.
2. Cut a transparent sheet in the shape of sector of a circle and fold it to obtain a right circular cone as shown in Fig.21.1.
3. Form 4 more such cones of the same size using transparent sheet. Put these cones on a hardboard.
4. Cut these cones with a transparent plane sheet in different positions as shown in Fig. 21.2 to Fig. 21.5.

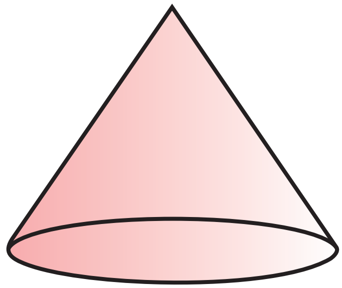

Fig 21.1

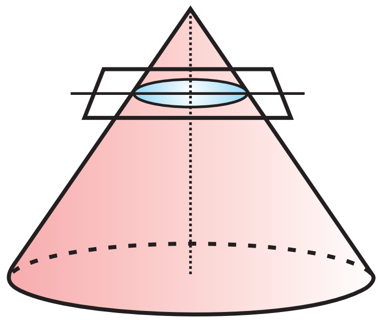

Fig 21.2

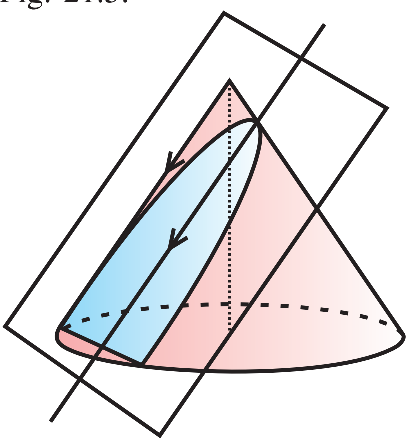

Fig 21.4

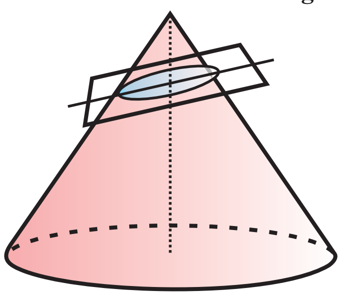

Fig 21.3

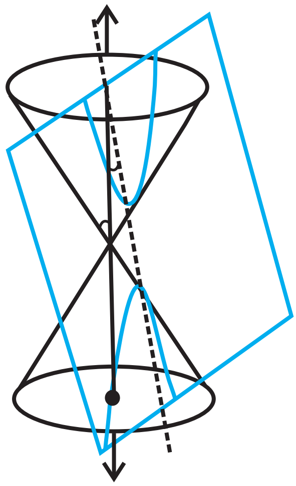

Fig 21.5

Demonstration

1. In Fig. 21.2, the transparent plane sheet cuts the cone in such a way that the sheet is parallel to the base of the cone. The section so obtained is a circle.
2. In Fig. 21.3, the plane sheet is inclined slightly to the axes of the cone. The section so obtained is an ellipse.
3. In Fig. 21.4, the plane sheet is parallel to a generator (slant height) of the cone. The section so obtained is a parabola.
4. In Fig. 21.5 the plane is parallel to the axis of the cone. The sections so obtained is a part of a hyperbola.

Observation

1. In Fig. 21.2, the transparent plane sheet is ______ to the base of the cone. The section obtained is _____.
2. In Fig. 21.3, the plane sheet is inclined to ______. The conic section obtained is _________.
3. In Fig. 21.4, the plane sheet is parallel to the _________. The conic section so obtained is ______.
4. In Fig. 21.5, the plane sheet is ________ to the axis. The conic section so obtained is a part of ______.

Application

This activity helps in understanding various types of conic sections which have wide spread applications in real life situations and modern sciences. For example, conics have interesting geometric properties that can be used for the reflection of light rays and beams of sound, i.e.

1. Circular disc reflects back the light issuing from centre to the centre again.
2. Elliptical disc reflects back the light issuing from one focus to the other focus.
3. Parabolic disc reflects back the light issuing from one focus parallel to its axis.
4. Hyperbolic disc reflects back the light issuing from one focus as if coming from other focus.

Activity 29

Objective

Verification of the geometrical significance of derivative.

Material Required

Graph sheets, adhesive, hardboard, trigonometric tables, geometry box, wires.

Method of Construction

1. Paste three graph sheets on a hardboard and draw two mutually perpendicular lines representing x-axis and y-axis on each of them.
2. Sketch the graph of the curve (circle) $x^2 + y^2 = 25$ on one sheet.
3. On the other two sheets sketch the graphs of $(x-3)^2 + y^2 = 25$ and the curve $xy = 4$ (rectangular hyperbola).

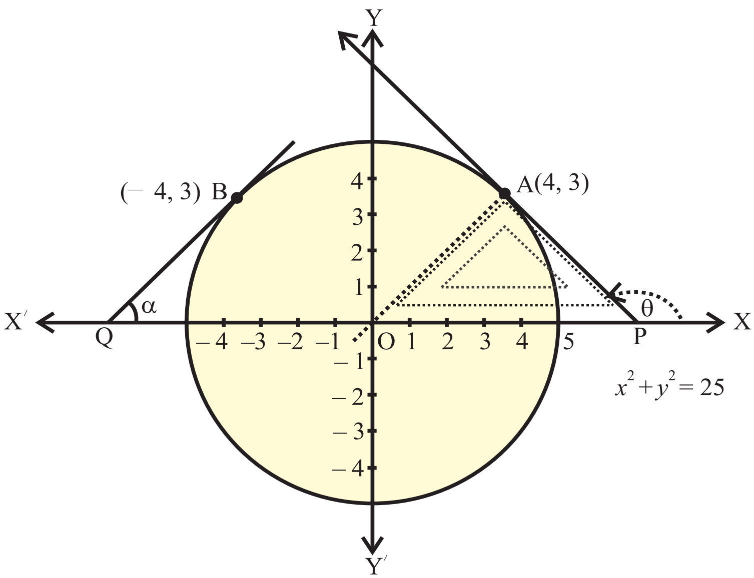

Fig. 29.1

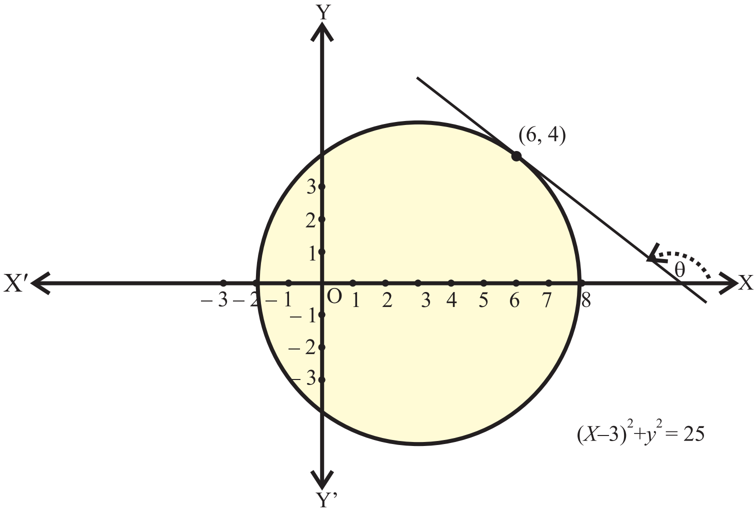

Fig 29.2

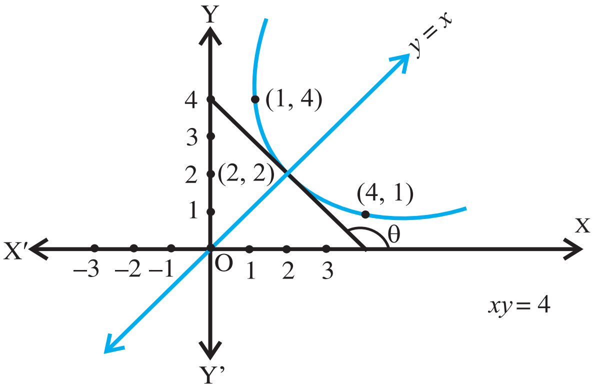

Fig. 29.3

DEMONSTRATION

1. Take first sheet on which, the graph of the circle $x^2 + y^2 = 25$ has been drawn (see Fig.29.1.
2. Take a point A (4, 3) on the circle.
3. With the help of a set square, place a wire in the direction OA and other perpendicular to OA at the point A to meet *x*-axis at a point (say P).

4. Measure the angle between the wire and the positive direction of x-axis at P (say θ).

5. Then find tan θ (with the help of trigonometric tables)

$$\text{Now, } x^2 + y^2 = 25 \Rightarrow y = \sqrt{25 - x^2} \Rightarrow \frac{dy}{dx} = \frac{-x}{\sqrt{25 - x^2}}.$$

Find $\frac{dy}{dx}$ at the point (4, 3) and verify that $\left(\frac{dy}{dx}\right)$ at (4, 3) = tan θ.

6. Similarly, take another point (– 4, 3) on the circle. Verify that $\frac{dy}{dx}$ at (– 4, 3) = tan α where α is the angle made by the tangent to the circle at the point (– 4, 3) with the positive direction of x-axis. (see Fig. 29.1).

7. Take other sheet with the graph of $(x – 3)^2 + y^2 = 25$ and take the point (6,4) on it and repeat the above process using set square and wires as shown in Fig. 29.2, i.e. verify that $\frac{dy}{dx}$ at (6, 4) = tan θ.

8. Now take the third sheet, showing the graph of the curve $xy = 4$. Take the point (2, 2) on it. Place one perpendicular side of set square along the line $y = x$ and a wire along the other side touching the curve at the point (2, 2) and find the angle made by the wire with the positive direction of x-axis as shown in Fig. 29.3. Let it be θ. Verify that $\frac{dy}{dx}$ at (2, 2) = tan θ.

Observation

1. For the curve $x^2 + y^2 = 25$, $\frac{dy}{dx}$ at the point (3, 4) = ________. Value of θ = _____ tan θ = ________ $\frac{dy}{dx}$ at (3, 4) = __________.

2. For the curve $x^2 + y^2 = 25$, $\frac{dy}{dx}$ at $(-4, 3)$ = ________, $\tan \alpha$ = ________

, $\frac{dy}{dx}$ at $(-4, 3)$ = ________.

3. For the curve $(x-3)^2 + y^2 = 25$, $\frac{dy}{dx}$ at $(6, 4)$ = _____ , value of θ = _______,

$\tan \theta$ = ______, $\frac{dy}{dx}$ at $(6, 4)$ = ________.

4. For the curve $xy = 4$, $\left(\frac{dy}{dx}\right)$ at $(2, 2)$ = ______ ,

θ = ____, $\tan \theta$ = ______.

APPLICATION

Same activity can be used to verify the result that the slope of the tangent at a point is equal to the value of the derivative at that point for other curves.

NOTE

The activity may be repeated by taking point (4, 3) on first sheet, (0, 4) on second sheet and (1, 4) on the third sheet.

Activity 33

Objective

To write the sample space, when a coin is tossed once, two times, three times, four times.

Material Required

One rupee coin, paper pencil/pen, plastic circular discs, marked with Head (H) and Tail (T).

Method of Construction

1. Toss a coin once. It can have two outcomes – Head or Tail.
2. Make a tree diagram showing the two branches of a tree - with H (Head) on one branch and T (Tail) on the other (see Fig. 33.1).
3. Write its sample space.
4. Toss a coin twice. It can have four outcomes (see Fig. 33.2)
5. Repeat the experiment with tossing the coin three times, four times, ..., *n* and write their sample spaces, if possible. (see Fig. 33.3 and 33.4).

Demonstration

1. If a coin is tossed once, the sample space is

$$S = \{H, T\}$$

Number of elements in $S = 2 = 2^1$

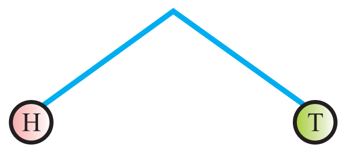

Fig. 33.1

2. When a coin is tossed twice, the sample space is

$$S = \{HH, HT, TH, TT\}$$

Number of elements in $S = 4 = 2^2$

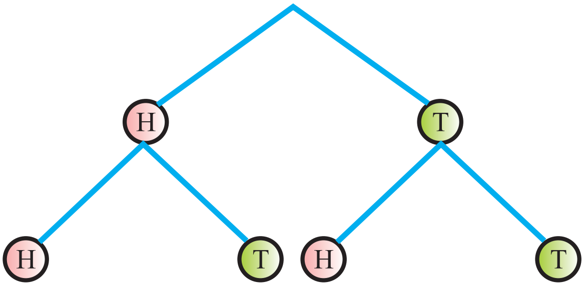

Fig. 33.2

3. When a coin is tossed three times, the sample space is

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

Number of elements in S = 8 = 2^3

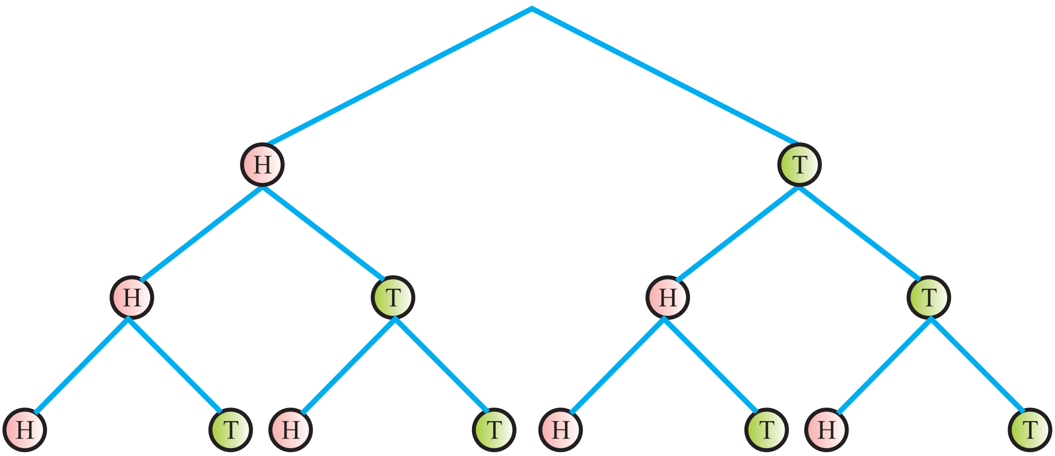

Fig. 33.3

4. When a coin is tossed four times, the S = Sample space is

$$\left\{\begin{array}{l}\text{HHHH, HHHT, HHTH, HHTT, HTHH, HTHT, HTTH, HTTT,}\\ \text{THHH, THHT, THTH, THTT, TTHH, TTHT, TTTH, TTTT}\end{array}\right\}$$

Number of elements in S = 16 = 2^4 and so on.

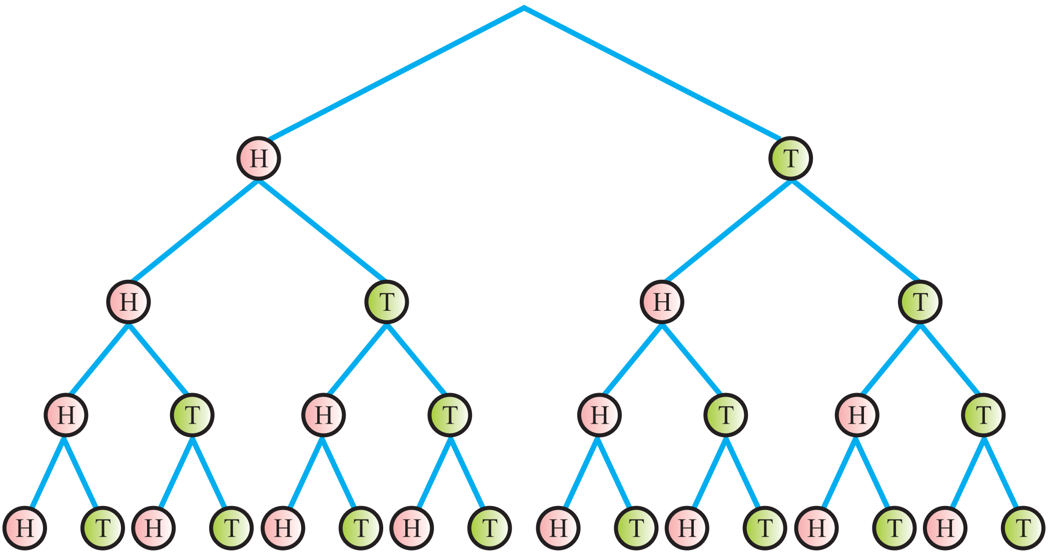

Fig. 33.4

OBSERVATION

Number of elements in sample space, when a

1. coin is tossed once = __________.
2. coin is tossed twice = __________.
3. coin is tossed three times = __________.
4. coin is tossed four times = __________.

APPLICATION

Sample space of an experiment is useful in determining the probabilities of different events associated with the sample space.