THREE DIMENSIONAL GEOMETRY **CM121101**

1. If a line makes angles α, β, γ with the positive direction of coordinate axes, then write the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$.

2. Write the vector equation of the line passing through (1, 2, 3) and perpendicular to the plane $\vec{r}.\left(\hat{\imath} + 2\hat{\jmath} - 5\hat{k}\right) + 9 = 0$.

3. Find the points on line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance of 5 units from the point P(1,3,3).

4. Show that the points whose position vectors are $-2\hat{\imath} + 3\hat{\jmath}, \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ and $7\hat{\imath} + 9\hat{k}$ are collinear.

5. Show that the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ intersect. Find their point of intersection.

6. Find foot of perpendicular from (0,2,7) from $\frac{x+2}{-1} = \frac{y-1}{3} = \frac{z-3}{-2}$.

7. Write the equation of the plane whose intercepts on the coordinate axes are $2, -3, 4$.

8. Write vector equations of the following lines : $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}, \frac{x-3}{4} = \frac{y-3}{6} = \frac{z+5}{12}$ and hence determine the distance between them.

9. Find the vector and cartesian equations of a line through the point (1, –1, 1) and perpendicular to the lines joining the points (4, 3, 2), (1, –1, 0) & (1, 2, –1), (2, 1, 1).

10. Find the shortest distance between the lines $\vec{r} = \hat{\imath} + 2\hat{\jmath} + \hat{k} + \lambda\left(\hat{\imath} - \hat{\jmath} + \hat{k}\right)$ and $\vec{r} = 2\hat{\imath} - \hat{\jmath} - \hat{k} + \mu\left(2\hat{\imath} + \hat{\jmath} + 2\hat{k}\right)$.

11. Find the shortest distance between the following lines : $\vec{r} = 2\hat{\imath} - 5\hat{\jmath} + \hat{k} + \lambda\left(3\hat{\imath} + 2\hat{\jmath} + 6\hat{k}\right)$ and $\vec{r} = 7\hat{\imath} - 6\hat{k} + \mu\left(\hat{\imath} + 2\hat{\jmath} + 2\hat{k}\right)$.

12. Find the shortest distance between the lines x+1=2y=–12z and x=y+2=6z–6.

13. Find the shortest distance between the lines l_1 and l_2, whose vector equations are : $\vec{r} = \hat{\imath} + \hat{\jmath} + \lambda\left(2\hat{\imath} - \hat{\jmath} + \hat{k}\right)$ and $\vec{r} = 2\hat{\imath} + \hat{\jmath} - \hat{k} + \mu\left(3\hat{\imath} - 5\hat{\jmath} + 2\hat{k}\right)$.

14. Find the distance between $\vec{r}.\left(2\hat{\imath} - \hat{\jmath} - 2\hat{k}\right) = 6$ & $\vec{r}.\left(6\hat{\imath} - 3\hat{\jmath} - 6\hat{k}\right) = 27$.

15. Find the equation of the plane passing through the points (–1, 2, 0), (2, 2, –1) and parallel to the line $\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$.

16. Find the equation of a line passing through the point (1, 2, – 4) and perpendicular to two lines $\vec{r} = \left(8\hat{\imath} - 19\hat{\jmath} + 10\hat{k}\right) + \lambda\left(3\hat{\imath} - 16\hat{\jmath} + 7\hat{k}\right)$ and $\vec{r} = \left(15\hat{\imath} + 29\hat{\jmath} + 5\hat{k}\right) + \mu\left(3\hat{\imath} + 8\hat{\jmath} - 5\hat{k}\right)$.

17. Find the value of k for which the following lines are perpendicular to each other :

$\frac{x+3}{k-5}=\frac{y-1}{1}=\frac{5-z}{-2k-1};\ \frac{x+2}{-1}=\frac{2-y}{-k}=\frac{z}{5}$.

18. Show that the lines $\frac{x+3}{-3}=\frac{y-1}{1}=\frac{z-5}{5}$ and $\frac{x+1}{-1}=\frac{y-2}{2}=\frac{z-5}{5}$ are coplanar.

19. Find the shortest distance between the lines $\vec{r}=3\hat{\imath}+2\hat{\jmath}-4\hat{k}+\lambda(\hat{\imath}+2\hat{\jmath}+2\hat{k})$ and $\vec{r}=5\hat{\imath}-2\hat{\jmath}+\mu(3\hat{\imath}+2\hat{\jmath}+6\hat{k})$, If the lines intersect find their point of intersection

20. Find the distance of the point $(-2,3,-4)$ from the line $\vec{r}=\hat{\imath}+2\hat{\jmath}-\hat{k}+\lambda(\hat{\imath}+3\hat{\jmath}-9\hat{k})$.