VECTOR ALGEBRA CM121001

1. Can a vector have direction angles $45^o, 60^o, 120^o$.
2. Find the direction cosines of the vector $4\hat{\imath} + 8\hat{\jmath} + \hat{k}$.
3. Find a vector $\vec{r}$ of magnitude $3\sqrt{2}$ units which makes angle of $\frac{\pi}{4}$ and $\frac{\pi}{2}$ with y and z-axes respectively.
4. Let $\vec{a} = 4\hat{\imath} + 5\hat{\jmath} - \hat{k}, \vec{b} = \hat{\imath} - 4\hat{\jmath} + 5\hat{k}$ and $\vec{c} = 3\hat{\imath} + \hat{\jmath} - \hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and satisfying $\vec{d}.\vec{c} = 21$.
5. Scalar product of the vector $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$ with a unit vector along the sum of the vectors $\vec{b} = 2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$ and $\vec{c} = \lambda\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ is equal to 1. Find the value of λ and hence find the unit vector along $\vec{b} + \vec{c}$.
6. Find the projection of $\vec{b} + \vec{c}$ on $\vec{a}$, where $\vec{a} = 2\hat{\imath} - 2\hat{\jmath} + \hat{k}, \vec{b} = \hat{\imath} + 2\hat{\jmath} - 2\hat{k}$ and $\vec{c} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k}$.
7. If $\vec{a}$ and $\vec{b}$ are two vectors of same magnitude inclined at an angle of 30^o such that $\vec{a}.\vec{b} = 3$, find $|\vec{a}|, |\vec{b}|$.
8. Decompose the vector $6\hat{\imath} - 3\hat{\jmath} - 6\hat{k}$ into vectors which are parallel and perpendicular to the vector $\hat{\imath} + \hat{\jmath} + \hat{k}$.
9. Let $\vec{a} = 5\hat{\imath} - \hat{\jmath} + 7\hat{k}$ and $\vec{b} = \hat{\imath} - \hat{\jmath} + \lambda\hat{k}$. Find λ such that $\vec{a} + \vec{b}$ is orthogonal to $\vec{a} - \vec{b}$.
10. Let $\vec{u}, \vec{v}$ and $\vec{w}$ be vectors such that $\vec{u} + \vec{v} + \vec{w} = \vec{0}$. If $|\vec{u}| = 3, |\vec{v}| = 4$ and $|\vec{w}| = 5$, then find $\vec{u}.\vec{v} + \vec{v}.\vec{w} + \vec{w}.\vec{u}$.
11. Let $\vec{a} = x^2\hat{\imath} + 2\hat{\jmath} - 2\hat{k}, \vec{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$ and $\vec{c} = x^2\hat{\imath} + 5\hat{\jmath} - 4\hat{k}$ be three vectors. Find the values of x for which the angle between $\vec{a}$ and $\vec{b}$ is acute and the angle between $\vec{b}$ and $\vec{c}$ is obtuse.
12. Find the angle between two vectors $\vec{a}$ and $\vec{b}$, if $|\vec{a} \times \vec{b}| = \vec{a}.\vec{b}$.
13. Find a unit vector perpendicular to plane ABC, where the coordinates of A, B and C are $A(3,-1,2), B(1,-1,-3)$ and $C(4,-3,1)$.
14. If $|\vec{a}| = \sqrt{26}, |\vec{b}| = 7$ and $|\vec{a} \times \vec{b}| = 35$, find $\vec{a}.\vec{b}$.
15. Let $\vec{a} = \hat{\imath} + 4\hat{\jmath} + 2\hat{k}, \vec{b} = 3\hat{\imath} - 2\hat{\jmath} + 7\hat{k}$ and $\vec{c} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}.\vec{d} = 15$.
16. Find a unit vector perpendicular to each of the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$, where $\vec{a} = 3\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ and $\vec{b} = \hat{\imath} + 2\hat{\jmath} - 2\hat{k}$.
17. Using vectors find the area of the triangle with vertices $A(2,3,5), B(3,5,8)$ and $C(2,7,8)$.
18. If $\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + \hat{k}, \vec{b} = -\hat{\imath} + \hat{k}, \vec{c} = 2\hat{\jmath} - \hat{k}$ are three vectors, find the area of the parallelogram having

diagonals $\vec{a}+\vec{b}$ and $\vec{b}+\vec{c}$.

19. The two adjacent sides of a parallelogram are $2\hat{\imath}-4\hat{\jmath}+5\hat{k}$ and $\hat{\imath}-2\hat{\jmath}+3\hat{k}$. Find the unit vector parallel to one of its diagonals. Also, find its area.

20. The two adjacent sides of a parallelogram are $2\hat{\imath}-4\hat{\jmath}-5\hat{k}$ and $2\hat{\imath}+2\hat{\jmath}+3\hat{k}$. Find the two unit vectors parallel to its diagonals. Using the diagonal vectors, find the area of the parallelogram.