APPLICATION OF DERIVATIVES **CM120601**

Q(1-10) carry 2 marks each.

1. The total cost C(x) associated with the production of x units of an item is given by $C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$. Find the marginal cost when 3 units are produced, where by marginal cost we mean the instantaneous rate of change of total cost at any level of output.

2. The volume of a sphere is increasing at the rate of 8 cm^3/s. Find the rate at which its surface area is increasing when the radius of the sphere is 12 cm.

3. The volume of a sphere is increasing at the rate of 3 cubic centimeter per second. Find the rate of increase of its surface area, when the radius is 2 cm.

4. Show that the function $f(x) = 4x^3 - 18x^2 + 27x - 7$ is always increasing on R.

5. The volume of a cube is increasing at the rate of 8 cm^3/s. How fast is the surface area increasing when the length of its edge is 12 cm?

6. Find the intervals in which the function $f(x) = \frac{x^4}{4} - x^3 - 5x^2 + 24x + 12$ is

(i) strictly increasing (ii) strictly decreasing.

7. Find the intervals in which the function $f(x) = -2x^3 - 9x^2 - 12x + 1$ is

(i) strictly increasing (ii) strictly decreasing

8. The length x of a rectangle is decreasing at the rate of 5 cm/min and the width y is increasing at the rate of 4 cm/min. When x = 8 cm and y = 6 cm, find the rate of change of

(i) the perimeter. (ii) area of rectangle.

9. Find the intervals in which the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is

(i) strictly increasing. (ii) strictly decreasing.

10. The sides of an equilateral triangle are increasing at the rate of 2 cm/s. Find the rate at which the area increases, when the side is 10 cm?

Q(11-16) carry 3 marks each.

11. Find the value(s) of x for which $y = [x(x - 2)]^2$ is an increasing function.

12. A ladder 5 m long is leaning against a wall. Bottom of ladder is pulled along the ground away from wall at the rate of 2 m/s. How fast is the height on the wall decreasing, when the foot of ladder is 4 m away from the wall?

13. Show that $y = \log(1 + x) \frac{2x}{2+x}$, $x > -1$ is an increasing function of x, throughout its domain.

14. Sand is pouring from the pipe at the rate of 12 cm3/s. The falling sand forms a cone on a ground in such a way that the height of cone is always one-sixth of radius of the base. How fast is the height of sand 5 cone increasing when the height is 4 cm?

15. Find the intervals in which the function f(x) = sin x + cos x, $0 \le x \le 2\pi$ is strictly increasing and strictly decreasing.

16. Prove that $y = \frac{4\sin\theta}{2+\cos\theta} - \theta$ is an increasing function in $(0, \frac{\pi}{2})$.

Q(17-25) carry 5 marks each.

17. An open tank with a square base and vertical sides is to be constructed from a metal sheet so as to hold a given quantity of water. Show that the cost of material will be least when depth of the tank is half of its width. If the cost is to he borne by nearby settled lower income families, for whom water will be provided.

18. The sum of the perimeters of a circle and square is k, where k is some constant. Prove that the sum of their areas is least, when the side of the square is double the radius of the circle.

19. Prove that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also, find the maximum volume.

20. Find the point on the curve $y^2 = 4x$, which is nearest to the point (2, – 8).

21. Show that the altitude of the right circular cone of maximum volume that can he inscribed in a sphere of radius r is $\frac{4r}{3}$.

22. A window is of the form of a semi-circle with a rectangle on its diameter. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.

23. Show that the surface area of a closed cuboid with square base and given volume is minimum, when it is a cube.

24. If the sum of lengths of the hypotenuse and a side of a right angled triangle is given, show that the area of the triangle is maximum, when the angle between them is $\frac{\pi}{3}$.

25. Prove that the least perimeter of an isosceles triangle in which a circle of radius r can be inscribed, is $6\sqrt{3}\,r$.