LIMITS AND DERIVATIVES CM111201

1. Evaluate $\lim_{x\to 2} f(x)$ (if it exists), where $f(x) = \begin{cases} x-[x], x<2 \\ 4, x=2 \\ 3x-5, x>2 \end{cases}$.

2. Evaluate $\lim_{x\to\sqrt{2}} \frac{x^2-2}{x^2+\sqrt{2}x-4}$.

3. Evaluate $\lim_{x\to 1}\left\{\frac{x-2}{x^2-x} - \frac{1}{x^3-3x^2+2x}\right\}$.

4. Evaluate $\lim_{x\to\sqrt{10}} \frac{\sqrt{7+2x}-(\sqrt{5}+\sqrt{2})}{x^2-10}$.

5. Evaluate $\lim_{x\to 0^-} \frac{x^2-3x+2}{x^3-2x^2}$.

6. Evaluate $\lim_{x\to 0} \frac{\sqrt{1+x^2}-\sqrt{1+x}}{\sqrt{1+x^3}-\sqrt{1+x}}$.

7. Evaluate $\lim_{x\to a} \frac{(x+2)^{\frac{5}{2}}-(a+2)^{\frac{5}{2}}}{x-a}$.

8. Evaluate $\lim_{x\to-\infty} \left(\sqrt{4x^2-7x}+2x\right)$.

9. Evaluate $\lim_{n\to\infty} \frac{(n+2)!+(n+1)!}{(n+2)!-(n+1)!}$.

10. Evaluate $\lim_{x\to 0} \frac{1-\cos x\sqrt{\cos 2x}}{x^2}$.

11. Evaluate $\lim_{h\to 0} \frac{(a+h)^2 \sin(a+h)-a^2 \sin a}{h}$.

12. Evaluate $\lim_{x\to a} \frac{\cos\sqrt{x}-\cos\sqrt{a}}{x-a}$.

13. Evaluate $\lim_{x\to\frac{\pi}{4}} \frac{\sqrt{\cos x}-\sqrt{\sin x}}{x-\frac{\pi}{4}}$.

14. Evaluate $\lim_{x\to\frac{\pi}{2}} \frac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x}$.

15. Evaluate $\lim_{x\to\pi} \frac{\sqrt{2+\cos x}-1}{(\pi-x)^2}$.

16. Differentiate $e^{\sqrt{ax+b}}$ with first principle method.

17. Differentiate $\sqrt{\tan x}$ with first principle method.

18. Differentiate $\tan\sqrt{x}$ with first principle method.

19. Differentiate $\log\left(\frac{1}{\sqrt{x}}\right) + 5x^a - 3a^x + \sqrt[3]{x^2} + 6\sqrt[4]{x^{-3}}$ w.r.t. x.

20. If $y = \sqrt{\frac{x}{a}} + \sqrt{\frac{a}{x}}$, prove that $2xy\frac{dy}{dx} = \left(\frac{x}{a} - \frac{a}{x}\right)$.

21. Differentiate $\frac{2^x \cot x}{\sqrt{x}}$ w.r.t. x.

22. Differentiate $log_{x^2} x$ w.r.t. x.

23. Differentiate $e^x \log \sqrt{x} \tan x$ w.r.t. x.

24. Differentiate $\frac{\sqrt{a}+\sqrt{x}}{\sqrt{a}-\sqrt{x}}$ w.r.t. x.

25. Differentiate $\frac{3^x}{x+\tan x}$ w.r.t. x.