

## Inverse Trigonometric Functions

**CM23M120201**

1. Find the domain of:

(i)  $f(x) = \sin^{-1} \sqrt{x^2 - 1}$

(ii)  $f(x) = 2 \cos^{-1} 2x + \sin^{-1} x$

(iii)  $\sec^{-1}(3x - 1)$

2. Evaluate:

(i)  $\sin^{-1} \left( -\frac{1}{2} \right) + 2 \cos^{-1} \left( -\frac{\sqrt{3}}{2} \right)$

(ii)  $\tan^{-1} \left( -\frac{1}{\sqrt{3}} \right) + \tan^{-1}(-\sqrt{3}) + \tan^{-1} \left( \sin \left( -\frac{\pi}{2} \right) \right)$

(iii)  $\operatorname{cosec}^{-1} \left( 2 \cos \frac{2\pi}{3} \right)$

(iv)  $\cot^{-1} \left\{ 2 \cos \left( \sin^{-1} \left( \frac{\sqrt{3}}{2} \right) \right) \right\}$

(v)  $\sin \left\{ \cos^{-1} \left( -\frac{3}{5} \right) + \cot^{-1} \left( -\frac{5}{12} \right) \right\}$

3. If  $(\sin^{-1} x)^2 + (\sin^{-1} y)^2 + (\sin^{-1} z)^2 = \frac{3}{4}\pi^2$ , find the value of  $x^2 + y^2 + z^2$ .

4. Write each of the following in simplest form:

(i)  $\tan^{-1} \left\{ \frac{\sqrt{1+x^2}-1}{x} \right\}, x \neq 0$

(ii)  $\sin \left\{ 2 \tan^{-1} \sqrt{\frac{1-x}{1+x}} \right\}$

5. Prove that:

(i)  $\tan \left( \sin^{-1} \frac{5}{13} + \cos^{-1} \frac{3}{5} \right) = \frac{63}{16}$

(ii)  $\frac{9\pi}{8} - \frac{9}{4} \sin^{-1} \frac{1}{3} = \frac{9}{4} \sin^{-1} \left( \frac{2\sqrt{2}}{3} \right)$

6. Solve:

(i)  $\cos \{2 \sin^{-1}(-x)\} = 0$

(ii)  $5 \tan^{-1} x + 3 \cot^{-1} x = 2\pi$

(iii)  $\sin^{-1} \frac{5}{x} + \sin^{-1} \frac{12}{x} = \frac{\pi}{2}$

(iv)  $\cos^{-1} \sqrt{3}x + \cos^{-1} x = \frac{\pi}{2}$

7. If  $\sin^{-1} x + \sin^{-1} y = \frac{\pi}{3}$  and  $\cos^{-1} x - \cos^{-1} y = \frac{\pi}{6}$ , find the values of x and y.

8. If  $(\sin^{-1} x)^2 + (\cos^{-1} x)^2 = \frac{17\pi^2}{36}$ , find x.

9. Solve:  $2 \tan^{-1}(\sin x) = \tan^{-1}(2 \sec x), x \neq \frac{\pi}{2}$ .

10. Solve:  $\tan^{-1}(x - 1) + \tan^{-1} x + \tan^{-1}(x + 1) = \tan^{-1} 3x$ .

11. Sum the series:  $\tan^{-1} \frac{1}{3} + \tan^{-1} \frac{2}{9} + \tan^{-1} \frac{4}{33} + \dots + \tan^{-1} \frac{2^{n-1}}{1+2^{2n-1}}$ .

12. If  $\sin^{-1} \frac{2a}{1+a^2} - \cos^{-1} \frac{1-b^2}{1+b^2} = \tan^{-1} \frac{2x}{1-x^2}$ , then prove that  $x = \frac{a-b}{1+ab}$ .

13. Show that  $2 \tan^{-1} x + \sin^{-1} \frac{2x}{1+x^2}$  is constant for  $x \geq 1$ , find that constant.

14. Prove that:  $2 \tan^{-1} \left( \sqrt{\frac{a-b}{a+b}} \tan \frac{\theta}{2} \right) = \cos^{-1} \left( \frac{a \cos \theta + b}{a+b \cos \theta} \right)$ .

15. For any a, b, x, y > 0, prove that:  $\frac{2}{3} \tan^{-1} \left( \frac{3ab^2-a^3}{b^3-3a^2b} \right) + \frac{2}{3} \tan^{-1} \left( \frac{3xy^2-x^3}{y^3-3x^2y} \right) = \tan^{-1} \left( \frac{2\alpha\beta}{\alpha^2+\beta^2} \right)$ , where  $\alpha = -ax + by, \beta = bx + ay$ .

