DETERMINANTS CM23M120401

1. Evaluate the following determinants :

(i) $\begin{vmatrix} 1 & 43 & 6 \\ 7 & 35 & 4 \\ 3 & 17 & 2 \end{vmatrix}$ (ii) $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$ (iii) $\begin{vmatrix} x+\lambda & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda \end{vmatrix}$ (iv) $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$

(v) $\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$ (vi) $\begin{vmatrix} a & b+c & a^2 \\ b & c+a & b^2 \\ c & a+b & c^2 \end{vmatrix}$ (vii) $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$ (viii) $\begin{vmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix}$

2. Using determinants, find k so that (k, 2–2k), (−k+1,2k) & (−4−k,6−2k) may be collinear.

3. If the points (x,– 2), (5,2), (8,8) are collinear, find x using determinants.

4. If the points (3,– 2), (x,2),(8,8) are collinear, find x using determinants.

5. Write thr minors and cofactors of the elements of the following determinants:

(i) $\begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}$ (ii) $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$.

6. Find the minors and cofactors of each element of the determinant $\begin{vmatrix} 2 & -2 & 3 \\ 1 & 4 & 5 \\ 2 & 1 & -3 \end{vmatrix}$.

7. If A= $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, find adj A.

8. If A= $\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, find adj A and verify that A(adj A) = (adj A)A=$|A|I_3$.

9. If A= $\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, show that $A^{-1} = \frac{1}{19}A$.

10. Show that A= $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ satisfies the equation $A^2 - 4A - 5I_3 = 0$ and hence, find A^{-1}.

11. Find the inverse of $\begin{bmatrix} a & b \\ c & \frac{1+bc}{a} \end{bmatrix}$.

12. For $A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 6 \\ 3 & 2 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$.

13. Find the inverse of the matrix A= $\begin{bmatrix} a & b \\ c & \frac{1+bc}{a} \end{bmatrix}$ and show that $aA^{-1} = (a^2+bc+1)I - aA$.

14. If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$, find the value of k so that $A^2 = kA - 2I$. Hence, Find A^{-1}.

15. Solve the following system of equations using matrix method.

(i) x+2y+z=7, x+3z=11, 2x-3y=1

(ii) $\frac{2}{x} - \frac{3}{y} + \frac{3}{z} = 10, \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 10, \frac{3}{x} - \frac{1}{y} + \frac{2}{z} = 13.$

(iii) x+y+z=3, 2x-y+z=-1, 2x+y–3z=-9.

(iv) $2x + y + z = 2, x + 3y - z = 5, 3x + y - 2z = 6$.

(v) $2x + 6y = 2,\ 3x - z = -8,\ 2x - y + z = -3$

(vi) $x - y + z = 2,\ 2x - y = 0,\ 2y - z = 1.$

(vii) $8x + 4y + 3z = 18,\ 2x + y + z = 5,\ x + 2y + z = 5.$

(viii) x+y+z=6, x+2z=7, 3x+y+z=12.

(ix) If A= $\begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$, find A^{-1} and hence solve the system of linear equations

3x+4y+7z=14, 2x-y+3z=4 and x+2y-3z=0.

(x) 2x–y+z =0, 3x+2y–z =0, x+4y+3z = 0